$\Delta M = \Delta p \cdot x$

$m' - m = (-1)\,80$

$= -80 + 1000$

$x = \dfrac{2(1000)}{5(5)} = 80$

$x' = \dfrac{2(920)}{5(4)} = 92$

$x = \dfrac{1000}{5(5)} = \dfrac{10000}{25} = 80$

$\Delta M = \Delta p \cdot x$

$m' - m = (-1)(80)$

$m' = m - 40$

$= 920$

$x' = \dfrac{920}{2(4)}$

$= \dfrac{920}{4}$

$= 230$

1. A consumer's budgetary allocation for two commodities x and y is given by m. Her demand for commodity x is given by: $x(p_x, p_y, m) = \dfrac{2m}{5p_x}$. Suppose that her budget allocation (m) and the price of commodity y (p_y) remains the same at (m = Rs. 1000, p_y = Rs. 20) while the price of commodity x (p_x) falls from Rs. 5 to Rs. 4. The *substitution effect* of this price change is given by

   ~~(A)~~ an increase in demand for x from 80 to 100

   (B) an increase in demand for x from 90 to 100

   ~~(C)~~ an increase in demand for x from 80 to 92

   (D) an increase in demand for x from 80 to 90

2. You are given the following partial information about the purchases of a consumer who consumes only two goods: Good 1 and Good 2.

	Year 1			Year 2	
	Quantity$_1$	Price$_1$		Quantity$_2$	Price$_2$
Good 1	100	100	Good 1	120	100
Good 2	100	100	Good 2	?? x	80

Suppose that the amount of Good 2 consumed in year 2 is denoted by x. Think about the range of x over which you would conclude that the consumer's consumption bundle in year 1 is *revealed preferred* to that in year 2. Also think about the range of x over which you would conclude that the consumer's consumption bundle in year 2 is *revealed preferred* to that in year 1. Which of the following ranges of x ensures that the consumer's behaviour is inconsistent (that is, it contradicts the *weak axiom of revealed preference*)?

(A) $x \leq 75$

(B) $x \geq 70$

(C) $70 < x < 75$

~~(D)~~ $75 < x < 80$

P_1 $\quad$ P_2

Q_1 $(100\times100, 100\times100) \geq 100\times100,\ 100\times80$

$\vee$

Q_2 $120\times100,\ x\times100$ $\geq$ $(120\times100, x\times80)$

$Q_1 \quad \geq \quad Q_2$

P_1 $\quad 2\times10{,}000 \geq 12{,}000 + 100x$

$Q_1 > Q_2 \quad 80<$

$(100+100) \ldots (120+x)$

$\dfrac{6000}{80} = 7.5$

P_2 $\quad 18{,}000 \geq 12000 + 80x$

$Q_2 > Q_1$

$\pi_i = pq - c(q) = (100 - 2(23)q)q - \frac{q^2}{2}$

$\pi' = 100 - 46 \times 2 q_i - q_i = 0.$

$q_i = \frac{100}{93}$

$MR = 100 - 2Q = 100 - 2(23)q$

$MC = q$

3. Consider a market demand function $p = 100 - \cancel{q}\ Q$, where p is market price and q is aggregate demand. There are 23 firms, each with cost function, $c_i(q_i) = \frac{q_i^2}{2}$, $i \in 1, 2, \ldots, 23$. The Cournot-Nash equilibrium

(A) involves each firm producing 3 units $p(+ve)$. 23×3

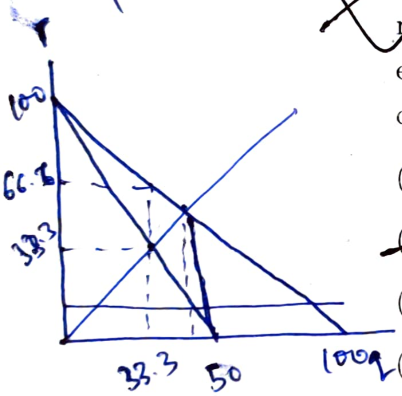
(B) involves each firm producing 4 units $p(+ve)$ 23×4

(C) involves each firm producing 5 units $p(-ve$ 23×5 d.

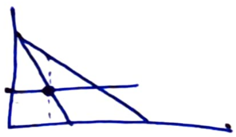
(D) is not well defined

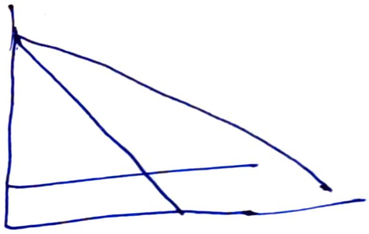
$100 - 2x = x$

4. Consider a market demand function $p = 100 - q$, where p is market price and q is aggregate demand. There are 10 firms, each with cost function, $c_i(q_i) = q_i$, $i \in 1, 2, \ldots, 10$. The firms compete in quantities. The total deadweight loss is



$MC_i = 1$





(A) $\frac{9^2}{2}$

~~(B)~~ $\cancel{\frac{99^2}{2}}$

(C) $\frac{10^2}{2}$

(D) $\frac{100^2}{2}$

5. Consider a market demand function $p = 100 - q$, where p is market price and q is aggregate demand. There is a large number of firms with identical cost functions

$$c_i(q_i) = \begin{cases} 10 + 2q_i, \text{ if } q_i > 0 \\ 0, \text{ otherwise.} \end{cases}$$

$MR = MC$

$100 - 2q = 1.$

$\frac{99}{2} = q$

$p = 100 - \frac{99}{2}$

$= \frac{101}{2}$

$MC = 2.$

~~(A)~~ The competitive equilibrium price is 2

(B) The competitive equilibrium price is 10

(C) The competitive equilibrium price is 2.1

(D) The competitive equilibrium price is not well defined

$100 - q = 1$

$q = 99$

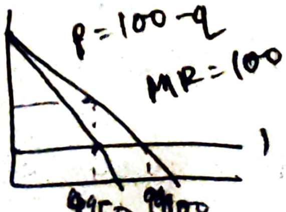
$(\frac{99}{2} - 1)$

$\times 50$

99×25

$\frac{1}{2} \times (99 - \frac{99}{2}) \times (100 - \frac{99}{2} - 1)$

$\frac{1}{2}(\frac{99}{2})^2$



$p = 100 - q$

$MR = 100 - 2q$

6. Consider a market demand function $p = 100 - q$, where p is market price and q is aggregate demand. There are two firms, firm 1 and firm 2, with identical cost functions

$$c_i(q_i) = \begin{cases} 0, \text{ if } q_i \leq 10 \\ \infty, \text{ otherwise,} \end{cases}$$

for $i = 1, 2$. The firms simultaneously announce their prices, p_1 and p_2. The demand coming to firm i is:

$$D_i(p_1, p_2) = \begin{cases} 100 - p_i, & \text{if } p_i < p_j \\ \frac{100-p_i}{2}, & \text{if } p_i = p_j \\ 0, & \text{otherwise.} \end{cases}$$

nash → same

The Bertrand-Nash equilibrium is

$10 = \frac{100 - p_i}{2}$

(A) $(p_1 = 0, p_2 = 0)$

(B) $(p_1 = 80, p_2 = 80)$

$100 - 20 = p_i$

(C) $(p_1 = 20, p_2 = 20)$

(D) $(p_1 = 90, p_2 = 90)$

7. 500 consumers (of health services) are distributed uniformly over the interval [0, 1]. The government can set up two hospitals anywhere in the interval. The hospitals provide health services free of cost, but the consumers have to incur the expenses of travelling to the hospital. The travel cost of a consumer who travels a distance d is d. The fixed cost of setting up a hospital is 300, and the marginal cost of servicing an individual is 2. The worth of the health services to an individual is 4. The government can, of course, decide to set up no hospital. The optimal hospital location decision of a welfare maximizing government is:

$C = 300 + 2n$

$W = (4 - 2 - d)500$

(A) set up no hospital $\Rightarrow W = 0$

(B) set up two hospitals – both at 1/2

(C) set up two hospitals – one at 1/4, the other at 3/4 $\quad \frac{5}{36} = \frac{1}{3}\times\frac{1}{3} + \frac{1}{6}\times\frac{1}{6}$

(D) set up two hospitals – one at 1/3, the other at 2/3 $\quad \frac{4.5}{36} = \frac{1}{8} = \frac{1}{4}\times\frac{1}{4} + \frac{1}{4}\times\frac{1}{4}$

0 1

8. There is a unit mass of consumers who buy either one unit of a product or nothing. Consumer valuation, θ, is distributed according to the distribution function $F(\theta)$ defined over $[\underline{\theta}, \overline{\theta}]$, that is, for any $\theta \in [\underline{\theta}, \overline{\theta}]$, the proportion of consumers with valuation less than or equal to θ is given by $F(\theta)$. Suppose that the inverse demand function for the product is $p(q)$, where p is market price and q is aggregate demand. Then the slope of the inverse demand function is

(A) $p'(q) = -\dfrac{1}{F'(p(q))}$

(B) $p'(q) = -F'(p(q))$

(C) $p'(q) = -\dfrac{1}{F'(q)}$

(D) $p'(q) = -F'(q)$

$p = f(q).$

$p = F(Q).$ $\quad p = F(Q).$

- **Questions 9 and 10 share the following common information.**

Consider an economy where output (income) is demand determined. In this economy λ proportion $(0 < \lambda < 1)$ of the total income is distributed to the workers, and $(1 - \lambda)$ proportion to the capitalists. The capitalists save s_c fraction $(0 < s_c < 1)$ of their income and consume the rest; the workers save s_w fraction $(0 < s_w < 1)$ of their income and consume the rest; also $s_w > s_c$. The aggregate demand consists of total consumption demand and total investment demand. Investment demand is autonomously given at $\overline{I}$ units.

9. Suppose savings propensities of both the workers and capitalists increase. Then, in the new equilibrium,

(A) aggregate savings increases and income decreases

(B) aggregate savings decreases and income increases

(C) aggregate savings remains unchanged and income decreases

(D) aggregate savings increases and income remains unchanged

$Y = AD.$

$Y = C + \overline{I} = \lambda(1 - s_w) + (1-\lambda)(1 - s_c).$

$\frac{\partial Y}{\partial \lambda} = 1 - s_w - 1(1 - s_c)$

$S = \lambda s_w + (1-\lambda) s_c$ $\quad s_w > s_c$ $\quad (1 - s_c) = (1 - s_w).$

10. Suppose savings propensities remain the same but the share of total income distributed to the workers increases. Then, in the new equilibrium,

(A) aggregate savings increases and income decreases

(B) aggregate savings decreases and income increases

(C) aggregate savings remains unchanged and income decreases

(D) aggregate savings increases and income remains unchanged

- **Questions 11, 12 and 13 are related and share a common information set.** The complete set of information is revealed gradually as you move from one question to the next. Attempt them sequentially starting from question 11.

11. Consider an economy where the aggregate output in the short run is given by $Y = (\overline{K})^{\alpha} L^{1-\alpha}$, $0 < \alpha < 1$, where L is the aggregate labour employment and $\overline{K}$ is the aggregate capital stock (which is fixed in the short run). Let P and W denote the aggregate price level and the nominal wage rate, respectively. The producers in the economy maximize profit in a perfectly competitive market.

In this economy the demand for labour as a function of real wage rate $(\frac{W}{P})$ is given by

(A) $L^d = Y^{\frac{1}{1-\alpha}} (\overline{K})^{\frac{\alpha}{1-\alpha}}$

(B) $L^d = \overline{K} (1-\alpha)^{\frac{1}{\alpha}} (\frac{W}{P})^{-\frac{1}{\alpha}}$

(C) $L^d = \overline{K} (1-\alpha)^{\frac{1}{\alpha}} (\frac{W}{P})^{\frac{1}{\alpha}}$

(D) $L^d = \overline{K} (1-\alpha)^{-\frac{1}{\alpha}} (\frac{W}{P})^{\frac{1}{\alpha}}$

$MP_L = \frac{w}{P}$

$\overline{K}^{\alpha} . L^{-\alpha} (1-\alpha) = \frac{w}{P}$

$\left(\frac{\overline{K}}{L}\right)^{\alpha} (1-\alpha) = \frac{w}{P}$

$L^{\alpha} = \overline{K}^{\alpha} (1-\alpha) . \frac{P}{w}$

$u = (wL^s)^\beta + (\bar{L} - L^s)^\beta$

$\frac{\partial u}{\partial L^s} = \beta \cdot (wL^s)^{\beta-1} \cdot w + \beta \cdot (\bar{L} - L^s)^{\beta-1} \cdot (-1) = 0.$

$w\beta(wL^s)^{\beta-1} = \beta \cdot (\bar{L} - L^s)^{\beta-1}$

$w^{\frac{1}{\beta-1}} \cdot wL^s = \bar{L} - L^s$

$w^{\frac{\beta}{\beta-1}} \cdot L^s = \bar{L} - L^s \qquad L^s = \frac{\bar{L}}{1 + \left(\frac{w}{P}\right)^{\beta/(\beta-1)}}$

12. The above economy is characterized by a representative household which takes the aggregate price level and the nominal wage rate as given and decides on its consumption and labour supply by maximizing its utility subject to its budget constraint. The household has a total endowment of $\bar{L}$ units of labour time, of which it supplies L^s units to the market and enjoys the rest as leisure. Its utility depends on its consumption (C) and leisure ($\bar{L} - L^s$) in the following way: $u = C^\beta + \left(\bar{L} - L^s\right)^\beta$, $0 < \beta < 1$. The only source of income of the household is the wage income and it spends its entire wage earning in buying consumption goods at the price P.

In this economy the supply of labour as a function of real wage rate $\left(\frac{W}{P}\right)$ is given by

(A) $L^s = \frac{\bar{L}}{1+\left(\frac{W}{P}\right)^{\frac{\beta}{\beta-1}}}$

~~(B)~~ $L^s = \frac{\bar{L}}{1+\left(\frac{W}{P}\right)^{\frac{\beta}{1-\beta}}}$

(C) $L^s = \bar{L}\left[1 + \left(\frac{W}{P}\right)^{\frac{\beta}{1-\beta}}\right]$

(D) $L^s = \frac{\bar{L}}{1-\left(\frac{W}{P}\right)^{\frac{\beta}{\beta-1}}}$

13. Given the labour demand and labour supply functions as derived above, the aggregate supply curve (output (Y) supplied as a function of the aggregate price level (P), with Y on x-axis and P on y-axis) of this economy is

(A) upward sloping

(B) downward sloping

(C) horizontal

(D) vertical

$L^s = L^d$

$\frac{\bar{L}}{1 + \left(\frac{w}{P}\right)^{\beta/(\beta-1)}} = \bar{K}(1-\alpha)^{\frac{1}{\alpha}}\left(\frac{w}{P}\right)^{-\frac{1}{\alpha}}$

$\frac{\bar{L}}{\bar{K}} \cdot (1-\alpha)^{-\frac{1}{\alpha}} = \left[1 + \left(\frac{w}{P}\right)^{\frac{\beta}{\beta-1}}\right]\left(\frac{w}{P}\right)$

$= Ly + Lr$

$-\frac{M}{P^2} \cdot dP = Ly + Lr$

$Sy \cdot dY + Sr \cdot dr = Ir \cdot dr$

$-\frac{M}{P^2} \cdot dP = Ly \cdot dY + Lr \cdot dr$

$dr = \frac{Sy \cdot dY}{Ir - Sr}$

$-\frac{M}{P^2} \cdot dP = Ly \cdot dY + Lr \cdot \frac{Sy \cdot dY}{Ir - Sr}$

$\frac{dP}{dY} = \frac{Ly + \frac{Lr Sy}{Ir - Sr}}{-\frac{M}{P^2}}$

14. Consider an economy with aggregate income Y and aggregate price level P. The goods market clearing condition is given by the savings-investment equality: $S(Y, r) = I(r)$, where r is real interest rate and $0 < S_Y < 1$, $S_r > 0$, $I_r < 0$. The money market clearing condition is given by the equality of real money supply ($\frac{M}{P}$) and demand for real balances (L): $\frac{M}{P} = L(Y, r)$, where M is the supply of money and $L_Y > 0$, $L_r < 0$. [For any function $f(x, y)$, f_x denotes the partial derivative of f with respect to x.]

The slope of the aggregate demand curve (aggregate output (Y) demanded as a function of the aggregate price level (P), with Y on x-axis and P on y-axis) of this economy is $\frac{dP}{dY}$.

(A) $\dfrac{S_Y L_r - (S_r - I_r) L_Y}{-\frac{M}{P^2} S_Y}$

~~(B)~~ $\dfrac{S_Y L_r - (S_r - I_r) L_Y}{\frac{M}{P^2} (S_r - I_r)}$

(C) $\dfrac{S_Y L_r - (S_r - I_r) L_Y}{-\frac{1}{P} S_Y}$

(D) $\dfrac{S_Y L_r - (S_r - I_r) L_Y}{\frac{1}{P} (S_r - I_r)}$

15. To test the prediction of the Solow growth model, you run the following linear regression for all the countries in the world:

$$g_i = \alpha + \beta_0 \log y_{i,0} + \beta_1 \log n_i + \beta_2 \log s_i + \gamma X_i + \varepsilon_i,$$

where g_i is the growth rate in per capita real GDP of country i over a certain period, $y_{i,0}$ is per capita real GDP of country i at the beginning of the period under consideration, n_i is population growth rate of country i, s_i is savings rate of country i, X_i stands for a set of other control variables and ε_i is the error term.

The Solow growth model predicts that the expected sign of the regression coefficient β_0 is

(A) positive

(B) negative

~~(C)~~ zero

(D) inconclusive

$f'(1) >$

$f'($

16. Consider the following matrix

$1(0-1) - 0 + 1(0-1)$

$-1 + -1 = -2.$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

The rank of A is

(A) 0

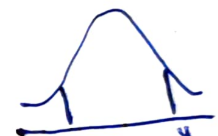
(B) 1

(C) 2

~~(D)~~ 3

19.

17. Bowl A contains two red coins; Bowl B contains two white coins; and Bowl C contains a white and a red coin. A bowl is selected uniformly at random and a coin is chosen from it uniformly at random. If the chosen coin is white, what is the probability that the other coin in the bowl is red?



$P(W) = \frac{1}{3}\left(0 + \frac{1}{2} + 1\right) = \frac{1}{2}$

$= \frac{1}{3} \times \frac{3}{2} =$

$P(R|W) = \dfrac{\frac{1}{3} \times \frac{1}{2}}{\frac{1}{2}}$

$\dfrac{\frac{1}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{3}{2}}$

~~(A)~~ $\frac{1}{3}$

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) $\frac{1}{6}$

20.

$f''(x) > 0$

$\dfrac{f'(1) - f'(\frac{1}{2})}{1 - \frac{1}{2}}$

$f'(1) > \dfrac{f(1) - f}{1 -}$

18. A girl chooses a number uniformly at random from $\{1, 2, 3, 4, 5, 6\}$. If she chooses n, then she chooses another number uniformly at random from $\{1, \ldots, n\}$. What is the probability that the second number is 5?

$\frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{5}$

~~(A)~~ $\frac{11}{180}$

(B) $\frac{2}{45}$

(C) $\frac{1}{3}$

(D) $\frac{1}{18}$

$\frac{1}{36} + \frac{1}{30}$

$\dfrac{30 + 36}{960}$

$\dfrac{30 \times 36}{6} = 180$

66

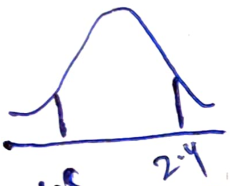
$f'(\frac{1}{2}) < \frac{f(1)-f(\frac{1}{2})}{1-\frac{1}{2}} = 1.$

19. The cumulative distribution function F of a standard normal distribution satisfies:

$$F(1.4) = 0.92, \;\; F(0.14) = 0.555, \;\; F(-0.2) = 0.42, \;\; F(-1.6) = 0.055$$

A manufacturer does not know the mean and standard deviation of the diameters of ball bearings it produces. However, he knows that the diameters follow a normal distribution with mean μ and standard deviation σ. It rejects 8% of bearings as too small if the diameter is less than 1.8 cm and 5.5% bearings as too large if the diameter is greater than 2.4 cm.

Which of the following is correct?



(1.4) < (2) → (1.6)

(A) $\mu = 2$

(B) $\mu = 2.33$

~~(C)~~ $\mu = 2.08$

(D) $\mu = 2.4$

$P(X > 2.4) = 0.055$

$P(X < 1.8) = 8\% = 0.92$

$= -1.4$

$\frac{X-\mu}{\sigma} > \frac{2.4-\mu}{\sigma} =$

$\boxed{1.6 = \frac{2.4-\mu}{\sigma}}$

$\frac{X-\mu}{\sigma} < \cancel{1.8} = -1.4$

$\frac{1.8-\mu}{\sigma} = +1.4$

$1.8-\mu = \cancel{+1.4\sigma}$

$\boxed{\frac{1.8-\mu}{\sigma} = -1.4}$

20. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function such that $f'(x)$ is strictly increasing in x ($f'(x)$ indicates the derivative of $f(x)$ with respect to x). Suppose $f(\frac{1}{2}) = \frac{1}{2}$ and $f(1) = 1$. Then which of the following is true ?

$f''(x) > 0$

$\frac{f'(1)-f'(\frac{1}{2})}{1-\frac{1}{2}}$

~~(A)~~ $f'(\frac{1}{2}) < 1 < f'(1)$

(B) $f'(\frac{1}{2}) < f'(1) < 1$

(C) $1 < f'(\frac{1}{2}) < f'(1)$

(D) None of the above

$f'(1) > \frac{f(1)-f(\frac{1}{2})}{1-\frac{1}{2}}$

$1.6\sigma + \mu = 2.4$

$-1.4\sigma + \mu = 1.8$

$+1.6\sigma + \mu = 2.4$

$-1.4\sigma + \mu = 1.8$

$+3\sigma = 0.8$

$\sigma = \frac{0.8}{3}$

$\mu = 2.4 - 1.6\sigma$

$= 2.4 - 1.6 \times \frac{0.8}{3}$

$16 \times 8 = 128$

$16 \times 6 = 96$

$= 2.4 - \frac{0.128}{3}$

~~0.04266~~

0.32

$(1-0)\frac{a^2}{2} + (\sqrt{2}-1)\frac{1^2}{2} + (\sqrt{3}-\sqrt{2})\frac{(\sqrt{2})^2}{2} + (\sqrt{4}-\sqrt{3})\frac{(\sqrt{3})^2}{2} + (\sqrt{5}-\sqrt{4})\frac{(\sqrt{4})^2}{2}$

$0 + \frac{\sqrt{2}-1}{2} + \frac{\sqrt{3}-\sqrt{2}}{1} + (2-\sqrt{3})\frac{3}{2} + (\sqrt{5}-2)\,2$

$\frac{\sqrt{2}}{2} - \frac{1}{2} + \sqrt{3}-\sqrt{2} + 3 - \frac{3\sqrt{3}}{2} + 2\sqrt{5} - 4$

21. For any non-negative real number x, define $f(x)$ to be the largest integer not greater than x. For instance, $f(1.2) = 1$. Evaluate the following integral

$\sqrt{2}-1 + \sqrt{8} - 2 + 2\sqrt{3} - 3 + (\sqrt{5}-2)\sqrt{4}$

$\sqrt{5} \approx 2.3\ldots$

$$\int_0^{\sqrt{5}} f(x^2)dx$$

$\int f(x)$ $\frac{x^2}{2}$

(A) 5

~~(B)~~ $(4\sqrt{5} - \sqrt{3} - \sqrt{2} - 3)(\frac{1}{2})$

(C) $4\sqrt{5}$

(D) $4(\sqrt{5} - 2)$

22. The constant term (i.e., the term not involving x) in the expansion of $\left(x + \frac{1}{x^2}\right)^{19}$ is

$^{19}C_0\, x^0 \left(\frac{1}{x^2}\right)^{19}$

(A) 1

(B) 19

$x^{13} \left(\frac{1}{x^2}\right)^{6}$

(C) 171

~~(D)~~ none of the above

no term exist.

23. Arjun and Gukesh each toss three different fair coins (each coin either lands heads or tails with equal probability and with each outcome independent of each other). Arjun wins if strictly more of his coins lands on heads than Gukesh, and we call the probability of this event p_1. Which of the following is correct?

$\times\frac{1}{8}(^3C_2 + ^3C_1 + ^3C_0)$

$\times\frac{1}{8}(^3C_2)(^3C_1 + ^3C_0)$

$\times\frac{1}{8}(^3C_1)(^3C_0)$

(A) $p_1 = \frac{1}{3}$

~~(B)~~ $p_1 = \frac{11}{32}$

(C) $p_1 = \frac{3}{8}$

(D) $p_1 = \frac{13}{32}$

$\frac{22}{64}$

$\frac{18}{64}$ $\frac{9}{32}$

3 2   2 1   1 0

3 1   2 0

3 0

$\frac{1}{8}(^3C_2 \cdot (\frac{1}{2})^3 + ^3C_1(\frac{1}{2})^3 + ^3C_0 \ldots$

$\frac{1}{8}\times\frac{1}{8}(3+3+1) = 7$

9

$(3+3+1)$

$+ (3)(3+1)$

$+ 3(1)$

$7 + 12 + 3$

$(^3C_2 \times (\frac{1}{8})^3)(2+1)\frac{1}{8}$

$(1-0)\frac{0^2}{2} + (\sqrt{2}-1)\frac{1}{2} + (\sqrt{3}-\sqrt{2})1 + (2-\sqrt{3})\frac{3}{2} + (\sqrt{5}-2)2$

$0 + \frac{\sqrt{2}}{2} - \frac{1}{2} + \sqrt{3} - \sqrt{2} + 3 - \frac{3\sqrt{3}}{2} + 2\sqrt{5} - 4$

$-\frac{\sqrt{2}}{2} - \frac{1}{2} - \frac{\sqrt{3}}{2} + 2\sqrt{5} - \frac{3}{2}$

$\frac{1}{2}(-\sqrt{2} - 3 - \sqrt{3} + 4\sqrt{5})$

24. How many real solutions are there to the equation $x|x| + 1 = 3|x|$?

(A) 0

(B) 1

(C) 2

~~(D) 3~~

$x<0$: $-x^2+1=-3x$, $x^2-3x-1=0$, $x=\frac{3\pm\sqrt{9+4}}{2}$, $\frac{3\pm\sqrt{13}}{2}$ 1 $x<0$

$x=0$: $1=0$ NP

$x>0$: $x^2+1=3x$, $x^2-3x+1=0$, 2 $x=\frac{3\pm\sqrt{9-4}}{2}$, $\frac{3\pm\sqrt{5}}{2}$

25. We are given n positive integers $k_1, \ldots, k_n$ (need not be distinct) such that

$$k_1 + \ldots + k_n = 5n - 4$$
$$\frac{1}{k_1} + \ldots + \frac{1}{k_n} = 1$$

What is the maximum value of n?

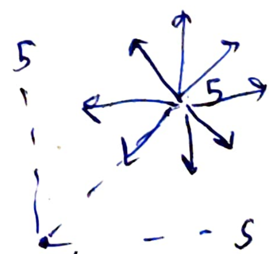
(A) 3

(B) 4

(C) 5

(D) 6

26. A monkey starts at the origin $(0,0)$ on $\mathbb{R}^2$. The monkey covers a distance of 5 units in any direction in one jump. If the monkey can only go to integer coordinates on $\mathbb{R}^2$, then the number of possible locations after its first jump is equal to



(A) 2

~~(B) 4~~

~~(C) 8~~

(D) 12

3 (4)
8

1 2 . . . . . n n+1 n+2

B $\underbrace{\qquad\qquad\qquad\qquad}_{\text{W (total=n).}}$ B

$(n-2)1 + \frac{2}{n}\left(\frac{1}{2}\right)$

$n-2+\frac{1}{n}$.

$P(B) = \frac{1}{2}\left(\frac{2}{n}\right) = \frac{1}{n}$

27. There is a strip made up of $(n+2)$ squares, where n is a positive integer. The two end squares are coloured black and other n squares are coloured white. A girl jumps to one of the n white squares uniformly at random and chooses one of its two adjacent squares uniformly at random. What is the probability that the chosen square is white?

(A) $1 - \frac{1}{n+2}$ (B) $1 - \frac{1}{n-1}$

~~(C)~~ $1 - \frac{1}{n}$ (D) $\frac{1}{2} - \frac{1}{n+1}$

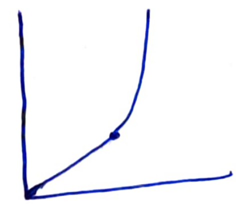
28. Let $f : \mathbb{R} \to \mathbb{R}$ be the following function

$$f(x) = \max(|x|, x^2), \qquad \forall\, x.$$

$\frac{(1+h)^2 - 1}{h} = \frac{1+h^2+2h-1}{h} = h+2$

$\frac{(1+h)-1}{-h}$

Which of the following is true?



(A) f is not continuous

~~(B)~~ f is continuous but not differentiable

(C) f is decreasing

~~(D)~~ f is increasing

dec

29. Let $f : \mathbb{R} \to \mathbb{R}$ be the following function

$$f(x) = \max(|x|, x^2), \qquad \forall\, x.$$

Define

$$D := \{(x, y) \in \mathbb{R}^2 : x \in \mathbb{R}, y \geq f(x)\}.$$

Which of the following is true for D?

(A) D is not convex

~~(B)~~ $\mathbb{R}^2 \setminus D$ is convex

(C) $\mathbb{R}^2_+ \setminus D$ is convex

(D) None of the above

convex

30. Suppose $f : [-1, 1] \to \mathbb{R}$ is a function such that

$$f(x) = \frac{2-x^2}{2} f\left(\frac{x^2}{2-x^2}\right), \qquad \forall\, x \in [-1, 1].$$

Then, $f(-1)$ is equal to

(A) -1

(B) 1

(C) 0

(D) $\frac{1}{2}$

$-1 = \frac{x^2}{2-x^2}$

$-2 + x^2 = x^2$

$-2 \neq 0$

$\frac{2-1}{2} f(1) = \frac{1}{2} f\left(\frac{x^2}{2-x^2}\right)$

$f(-1) = \frac{2-1}{2} = \frac{1}{2} f\left(\frac{1}{2-1}\right)$

$f(1) = \frac{1}{3} f(\ldots)$

$f\left(\frac{1}{2}\right) = \frac{2-\frac{1}{4}}{2} f\left(\frac{\frac{1}{4}}{2-\frac{1}{4}}\right)$

$f\left(\frac{\frac{1}{4}}{\frac{7}{4}}\right)$

$\frac{1}{2-1} = \frac{1}{2}$

$f\left(\frac{1}{2}\right) = \frac{2-\frac{1}{4}}{2}$

$= \frac{\frac{7}{4}}{2}$

$= \frac{7}{8} f(-1)$

$\frac{x^2}{2-x} = -1$

$f\left(\frac{1}{2}\right) = \frac{7}{8} f(-1)$

$f(-1) = \frac{1}{2} f\left(\frac{1}{2}\right)$

$f(-1) = \frac{7}{8} \cdot \frac{1}{2} \cdot f(-1)$