SYLLABUS FOR MSQE
(Program Code: MQEK and MQED) 2016

Syllabus for PEA (Mathematics), 2016

Algebra: Binomial Theorem, AP, GP, HP, Exponential, Logarithmic Series, Sequence, Permutations and Combinations, Theory of Polynomial Equations; (up to third degree).

Matrix Algebra: Vectors and Matrices, Matrix Operations, Determinants.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables. Unconstrained Optimization, Definite and Indefinite Integrals: Integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Elementary Statistics: Elementary probability theory, measures of central tendency, dispersion, correlation and regression, probability distributions, standard distributions-Binomial and Normal.

Syllabus for PEB (Economics), 2016

Microeconomics: Theory of consumer behaviour, theory of production, market structure under perfect competition, monopoly, price discrimination, duopoly with Cournot and Bertrand competition (elementary problems) and welfare economics.

Macroeconomics: National income accounting, simple Keynesian Model of income determination and the multiplier, IS-LM Model, models of aggregate demand and aggregate supply, Harrod-Domar and Solow models of growth, money, banking and inflation.

PEA 2016 (Mathematics)

Answer all questions

1. Consider the polynomial $P(x) = ax^3 + bx^2 + cx + d$, where $a, b, c, d \in \{1, 2, \ldots, 9\}$. If $P(10) = 5861$, then the value of c is

   (a) 1.

   (b) 2.

   (c) 6.

   (d) 5.

2. Let $A \subset \mathbb{R}$, $f : A \to \mathbb{R}$ be a twice continuously differentiable function, and $x^* \in A$ be such that $\dfrac{\partial f}{\partial x}(x^*) = 0$.

   (a) $\dfrac{\partial^2 f}{\partial x^2}(x^*) \leq 0$ is a *sufficient* condition for x^* to be a point of local maximum of f on A;

   (b) $\dfrac{\partial^2 f}{\partial x^2}(x^*) \leq 0$ is a *necessary* condition for x^* to be a point of local maximum of f on A;

   (c) $\dfrac{\partial^2 f}{\partial x^2}(x^*) \leq 0$ is *necessary and sufficient* for x^* to be a point of local maximum of f on A;

   (d) $\dfrac{\partial^2 f}{\partial x^2}(x^*) \leq 0$ is *neither necessary nor sufficient* for x^* to be a point of local maximum of f on A.

3. You are given five observations x_1, x_2, x_3, x_4, x_5 on a variable x, ordered from lowest to highest. Suppose x_5 is increased. Then,

   (a) The mean, median, and variance, all increase.

   (b) The median and the variance increase but the mean is unchanged.

   (c) The variance increases but the mean and the median are unchanged.

(d) None of the above.

4. Suppose the sum of coefficients in the expansion $(x+y)^n$ is 4096. The largest coefficient in the expansion is:

   (a) 924.

   (b) 1024.

   (c) 824.

   (d) 724.

5. There are three cards. The first is green on both sides, the second is red on both sides and the third is green on one side and red on the other. I choose a card with equal probability, then a side of that card with equal probability. If the side I choose of the card is green, what is the probability that the other side is green?

   (a) $\frac{1}{3}$.

   (b) $\frac{1}{2}$.

   (c) $\frac{2}{3}$.

   (d) $\frac{3}{4}$.

6. The value of

$$\int_0^{\frac{\pi}{2}} x \sin x dx$$

   is:

   (a) 0.

   (b) -1.

   (c) $\frac{1}{2}$.

   (d) 1

7. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as follows:

$$f(x) = \begin{cases} ax + b & \text{if } x \geq 0 \\ \sin 2x & \text{if } x < 0 \end{cases}$$

For what values of a and b is f continuous but *not* differentiable?

(a) $a = 2$, $b = 0$.

(b) $a = 2$, $b = 1$.

(c) $a = 1$, $b = 1$.

(d) $a = 1$, $b = 0$.

8. A student wished to regress household food consumption on household income. By mistake the student regressed household income on household food consumption and found R^2 to be 0.35. The R^2 in the correct regression of household food consumption on household income is

(a) 0.65.

(b) 0.35.

c) $1 - (.35)^2$.

(d) None of the above.

9. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) = 3xe^y - x^3 - e^{3y}$$

.

Which of the following statements is true?

(a) $(x = 1, y = 0)$ is a local maximum of f.

(b) $(x = 1, y = 0)$ is a local minimum of f.

(c) $(x = 1, y = 0)$ is neither a local maximum nor a local minimum of f.

(d) $(x = 0, y = 0)$ is a global maximum of f.

10. Let

$$f(x) = \frac{x + \sqrt{3}}{1 - \sqrt{3}x}$$

for all $x \neq \frac{1}{\sqrt{3}}$. What is the value of $f(f(x))$?

(a) $\frac{x-\sqrt{3}}{1+\sqrt{3}x}$.

(b) $\frac{x^2+2\sqrt{3}x+3}{1-2\sqrt{3}x+3x}$.

(c) $\frac{x+\sqrt{3}}{1-\sqrt{3}x}$.

(d) $\frac{x+\sqrt{3}}{1-\sqrt{3}x}$.

11. The continuous random variable X has probability density $f(x)$ where

$$f(x) = \begin{cases} a & \text{if } 0 \leq x < k \\ b & \text{if } k \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

where $a > b > 0$ and $0 < k < 1$. Then $E(X)$ is given by:

(a) $\frac{b(1-a)^2}{2a(a-b)}$.

(b) $\frac{1}{2}$.

(c) $\frac{a-b}{(a+b)}$.

(d) $\frac{1-2b+ab}{2(a-b)}$.

12. The set of values of x for which $x^2 - 3|x| + 2 < 0$ is given by:

(a) $\{x : x < -2\} \cup \{x : x > 1\}$.

(b) $\{x : -2 < x < -1\} \cup \{x : 1 < x < 2\}$.

(c) $\{x : x < -1\} \cup \{x : x > 2\}$.

(d) None of the above.

13. The system of linear equations

$$\begin{aligned} (4d-1)x + y + z &= 0 \\ -y + z &= 0 \\ (4d-1)z &= 0 \end{aligned}$$

has a non-zero solution if:

(a) $d = \frac{1}{4}$.

(b) $d = 0$.

(c) $d \neq \frac{1}{4}$.

(d) $d = 1$.

14. Suppose F is a cumulative distribution function of a random variable x distributed in $[0, 1]$ defined as follows:

$$F(x) = \begin{cases} ax + b & \text{if } x \geq a \\ x^2 - x + 1 & \text{otherwise} \end{cases}$$

where $a \in (0, 1)$ and b is a real number. Which of the following is true?

(a) F is continuous in $(0, 1)$.

(b) F is differentiable in $(0, 1)$.

(c) F is not continuous at $x = a$.

(d) None of the above.

15. The solution of the optimization problem

$$\max_{x,y} \; 3xy - y^3$$

subject to

$$2x + 5y \geq 20$$

$$x - 2y = 5$$

$$x, y \geq 0.$$

is given by:

(a) $x = 19, y = 7$.

(b) $x = 45, y = 20$.

(c) $x = 15$, $y = 5$.

(d) None of the above.

16. Let $f : \mathbb{R} \to \mathbb{R}$ be a strictly increasing function. Let g be the inverse of the function f. If $f'(1) = g(1) = 1$, then $g'(1)$ equals to

(a) 0.

(b) $\frac{1}{2}$.

(c) -1.

(d) 1.

17. Consider a quadratic polynomial $P(x)$. Suppose $P(1) = -3, P(-1) = -9, P(-2) = 0$. Then, which of the following is true.

(a) $P(\frac{1}{2}) = 0$.

(b) $P(\frac{5}{2}) = 0$.

(c) $P(\frac{5}{4}) = 0$.

(d) $P(\frac{3}{4}) = 0$.

18. For any positive integers k, ℓ with $k \geq \ell$, let $C(k, \ell)$ denote the number of ways in which ℓ distinct objects can be chosen from k objects. Consider $n \geq 3$ distinct points on a circle and join every pair of points by a line segment. If we pick three of these line segments uniformly at random, what is the probability that we choose a triangle?

    (a) $\frac{C(n,2)}{C(C(n,2),3)}$.

    (b) $\frac{C(n,3)}{C(C(n,2),3)}$

    (c) $\frac{2}{n-1}$.

    (d) $\frac{C(n,3)}{C(C(n,2),2)}$.

19. Let $X = \{(x, y) \in \mathbb{R}^2 : x + y \leq 1, 2x + \frac{y}{2} \leq 1, x \geq 0, y \geq 0\}$. Consider the optimization problem of maximizing a function $f(x) = ax + by$, where a, b are real numbers, subject to the constraint that $(x, y) \in X$. Which of the following is not an optimal value of f for any value of a and b?

    (a) $x = 0, y = 1$.

    (b) $x = \frac{1}{3}, y = \frac{2}{3}$.

    (c) $x = \frac{1}{4}, y = \frac{1}{4}$.

    (d) $x = \frac{1}{2}, y = 0$.

20. Let $F : [0, 1] \to \mathbb{R}$ be a differentiable function such that its derivative $F'(x)$ is increasing in x. Which of the following is true for every $x, y \in [0, 1]$ with $x > y$?

    (a) $F(x) - F(y) = (x - y)F'(x)$.

    (b) $F(x) - F(y) \geq (x - y)F'(x)$.

    (c) $F(x) - F(y) \leq (x - y)F'(x)$.

    (d) $F(x) - F(y) = F'(x) - F'(y)$.

21. A bag contains N balls of which a $(a < N)$ are red. Two balls are drawn from the bag without replacement. Let p_1 denote the probability that the first ball is red and p_2 the probability that the second ball is red. Which of the following statements is true?

(a) $p_1 > p_2$.

(b) $p_1 < p_2$.

(c) $p_2 = \frac{a-1}{N-1}$.

(d) $p_2 = \frac{a}{N}$.

22. Let $t = x + \sqrt{x^2 + 2bx + c}$ where $b^2 > c$. Which of the following statements is true?

(a) $\frac{dx}{dt} = \frac{t-x}{t+b}$.

(b) $\frac{dx}{dt} = \frac{t+2x}{2t+b}$.

(c) $\frac{dx}{dt} = \frac{1}{2x+b}$.

(d) None of the above.

23. Let A be an $n \times n$ matrix whose entry on the i-th row and j-th column is $\min(i, j)$. The determinant of A is:

(a) n.

(b) 1.

(c) $n!$

(d) 0.

24. What is the number of non-negative integer solutions of the equation $x_1 + x_2 + x_3 = 10$?

(a) 66.

(b) 55.

(c) 100.

(d) None of the above.

25. The value of

$$\int_{b}^{2b} \frac{xdx}{x^2+b^2} \ ,$$

$b > 0$ is:

(a) $\frac{1}{b}$.

(b) $\ln 4b^2$.

(c) $\frac{1}{2}\ln(\frac{5}{2})$.

(d) None of the above.

26. Let f and g be functions on $\mathbb{R}^2$ defined respectively by

$$f(x,y) = \tfrac{1}{3}x^3 - \tfrac{3}{2}y^2 + 2x,$$

and

$$g(x,y) = x - y.$$

Consider the problems of *maximizing* and *minimizing* f on the constraint set $C = \{(x,y) \in \mathbb{R}^2 : g(x,y) = 0\}$.

(a) f has a maximum at $(x = 1, y = 1)$, and a minimum at $(x = 2, y = 2)$.

(b) f has a maximum at $(x = 1, y = 1)$, but does not have a minimum.

(c) f has a minimum at $(x = 2, y = 2)$, but does not have a maximum.

(d) f has neither a maximum nor a minimum.

27. A particular men's competition has an unlimited number of rounds. In each round, every participant has to complete a task. The probability of a participant completing the task in a round is p. If a participant fails to complete the task in a round, he is eliminated from the competition. He participates in every round before being eliminated. The competition begins with three participants. The probability that all three participants are eliminated in the same round is:

(a) $\frac{(1-p)^3}{1-p^3}$.

(b) $\frac{1}{3}(1-p)$.

(c) $\frac{1}{p^3}$.

(d) None of the above.

28. Three married couples sit down at a round table at which there are six chairs. All of the possible seating arrangements of the six people are equally likely. The probability that each husband sits next to his wife is:

(a) $\frac{2}{15}$.

(b) $\frac{1}{3}$.

(c) $\frac{4}{15}$.

(d) None of the above.

29. Let $f : \mathbf{R}^2 \to \mathbf{R}$ be a function. For every $x, y, z \in \mathbf{R}$, we know that $f(x,y) + f(y,z) + f(z,x) = 0$. Then, for every $x, y \in \mathbf{R}^2$, $f(x,y) - f(x,0) + f(y,0) =$

(a) 0.

(b) 1.

(c) -1.

(d) None of the above.

30. The minimum value of the expression below for $x > 0$ is:

$$\frac{\left(x+\frac{1}{x}\right)^6 - \left(x^6+\frac{1}{x^6}\right) - 2}{\left(x+\frac{1}{x}\right)^3 + \left(x^3+\frac{1}{x^3}\right)}$$

(a) 1.

(b) 3.

(c) 6

(d) 12.

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PEB (2016)
Answer any 6 questions. All questions carry equal marks.

1. Consider an exchange economy consisting of two individuals 1 and 2, and two goods, X and Y. The utility function of individual 1 is $U_1 = X_1 + Y_1$, and that of individual 2 is $min\{X_2, Y_2\}$, where X_i (resp. Y_i) is the amount of X (resp. Y) consumed by individual i, where $i = 1, 2$. Individual 1 has 4 units of X and 8 units of Y, and individual 2 has 6 units of X and 4 units of Y to begin with.

(i) What is the set of Pareto optimal outcomes in this economy? Justify your answer.

(ii) What is the competitive equilibrium in this economy? Justify your answer.

(iii) Are the perfectly competitive equilibria Pareto optimal?

(iv) Now consider another economy where everything is as before, apart from individual 2's preferences, which are as follows: (a) among any two any bundles consisting of X and Y, individual 2 prefers the bundle which has a larger amount of commodity X irrespective of the amount of commodity Y in the two bundles, and (b) between any two bundles with the same amount of X, she prefers the one with a larger amount of Y. Find the set of Pareto optimal outcomes in this economy. [6]+[6]+[2]+[6]

2. Consider a monopolist who can sell in the domestic market, as well as in the export market. In the domestic market she faces a demand $p_d = 10 - q_d$, where p_d and q_d are domestic price and demand respectively. In the export market she can sell unlimited quantities at a price of 4. Suppose the monopolist has a single plant with cost function $\frac{q^2}{4}$.

(a) Solve for total output, domestic sale and exports of the monopolist.

(b) Solve for the domestic and world welfare at this equilibrium. [10]+[10]

3. A consumer consumes electricity, denoted by E, and butter, denoted by B. The per unit price of B is 1. To consume electricity the consumer has to pay a fixed charge R, and a per unit price of p. If consumption of $E \leq \frac{1}{2}$ then $p = 1$; otherwise $p = 2$. The utility function of the consumer is $3E + B$, and her income is $I > R$.

(i) Draw the consumer's budget line.

(ii) If $R = 0$ and $I = 1$, find the consumer's optimal consumption of E and B.

(iii) Consider a different pricing scheme where there is a rental charge of R and the price of E is 1 for any $X \leq 1/2$, and every *additional* unit beyond $\frac{1}{2}$ is priced at $p = 2$. Find the optimal consumption of B and E when $R = 1$ and $I = 3$. [7]+[7]+[6]

4. A monopoly publishing house publishes a magazine, earning revenue from selling the magazine, as well as by publishing advertisements. Thus $R = q.p(q) + A(q)$, where R is total revenue, q denotes quantity, $p(q)$ is the inverse demand function, and $A(q)$ is the advertising revenue. Assume that $p(q)$ is decreasing and $A(q)$ is increasing in q. The cost of production $c(q)$ is also increasing in the quantity sold. Assume all functions are twice differentiable in q.

(i) Derive the profit-maximising outcome.

(ii) Is the marginal revenue curve necessarily negatively sloped?

(iii) Can the monopolist fix the price of the magazine below the marginal cost of production?[7]+[7]+[6]

5. Consider a Solow style growth model where the production function is given by

$$Y_t = A_t F(K_t, H_t)$$

where $Y_t =$ output of the final good, K_t is the capital stock, $A_t =$ the level of technology, and $H_t =$ the quantity of labor used in production (the labor force). Assume technology is equal to $A_t = A_0(1+\alpha)^t$ where $\alpha > 0$ is the growth rate of technology, A_0 is the time 0 level of technology, and $H_{t+1} = (1+n)H_t$, where $n > 0$ is the labour force growth rate. The production function is homogenous of degree 1 and satisfies the usual properties. (Assume that inputs are essential and Inada conditions hold). Assume that capital evolves according to

$$K_{t+1} = (1-\delta)K_t + I_t$$

where I_t is the level of investment.

(i) Define $y_t = \frac{Y_t}{H_t}$.Show that

$$y_t = A_t f(k_t)$$

where $f(k) = F(k, 1)$.

(ii) Define $k_t = \frac{K_t}{H_t}$ and $i_t = \frac{I_t}{H_t}$. Show that

$$k_{t+1} = \frac{(1-\delta)k_t + i_t}{1+n}$$

(iii) Suppose the savings rate is given by $s_t = \sigma y_t$ where $\sigma \in [0, 1]$. Derive the condition that determines the steady state capital stock when $\alpha = 0$. How many non-zero steady states are there ?

(iv) Let $\gamma_t = \frac{k_{t+1}}{k_t}$ be the gross growth rate. Suppose $\alpha = 0$. Derive an expression for γ_t and evaluate and discuss the sign for $\frac{d\gamma_t}{dk_t}$.

(v) Let $f(k_t) = k_t^\theta$, $A_0 = 1$, and $\alpha > 0$. Along a balanced growth path show that $\frac{k_{t+1}}{k_t}$ and $\frac{y_{t+1}}{y_t}$ grow at the same rate. [2]+[3]+[5]+[5]+[5]

6. Consider the aggregate supply curve for an economy given by

$$P_t = P_t^e(1+\mu)F(u_t, z)$$

where P_t = actual price level at time period t, P_t^e = expected prices at time t, and the function, F, given by,

$$F(u_t, z) = 1 - \alpha u_t + z$$

captures the effects of the unemployment rate (u_t) at time t and the level of unemploment benefits (z) on the price level (through their effects on wages). Assume $\mu > 0$ denotes the monopoly markup. Assume μ and z are constant.

(i) Show that the aggregate supply curve can be transformed to be written in terms of π_t (the inflation rate) and the expected inflation rate, π_t^e, i.e. $\pi_t = \pi_t^e + (\mu + z) - \alpha u_t$, where $\pi_t = \frac{P_{t-1} - P_t}{P_t}$ and $\pi_t^e = \frac{P_{t+1}^e - P_t}{P_t}$. What is this equation called ? Briefly interpret it.

(ii) Now assume that $\pi_t^e = \theta \pi_{t-1}$ where $\theta > 0$. What is this equation called ? Re-write the equation in the above bullet and interpret when $\theta = 1$ and $\theta \neq 1$.

(iii) Let $\pi_t^e = \pi_{t-1}$ Derive the natura rate of unemployment, and express the change in the inflation rate in terms of the natural rate. Briefly interpret this equation.

(iv) How would you think about wage indexation in this model ? Does wage indexation increase the effect of unemployment on inflation? Assume $\pi_t^e = \pi_{t-1}$. [8]+[3]+[6]+[6]

7. Consider an inter-temporal choice problem in which a consumer maximises utility,

$$U(c_1, c_2) = u(c_1) + \frac{u(c_2)}{1+\delta},$$

where c_i is the consumption in period i, $i = 1, 2$, and δ is the discount factor (measure of the consumer's impatience), subject to

$$c_1 + \frac{c_2}{1+\delta} = Y_1 + \frac{Y_2}{1+r} \equiv W,$$

where Y_i is the consumer's income in period $i = 1, 2$, and r is the rate of interest. Assume $c_i > 1\ \forall i$.

(i) Let $u(c_i) = log(c_i)$. Find a condition such that there is consumption smoothing.

(ii) Plot the two cases where (a) the consumer biases its consumption towards the future, and (b) where the consumer biases it consumption towards the present. Put c_2 on the vertical axis and c_1 on the horizontal axis.

(iii) Suppose there is consumption smoothing. Solve for $c_1^* = c_1(r, Y_1, Y_2)$. Interpret this equation.

(iv) Define Y_P, the permanent income, as that constant stream of income (Y_P, Y_P) which gives the same lifetime income as does the fluctuating income stream (Y_1, Y_2). What does this imply about the optimal choice of c_1, c_2, and Y_P? Interpret your result graphically. [5]+[6]+[4]+[5]

8. Consider a cake of size 1 which can be divided between two individuals, A and B. Let α (resp. β) be the amount allocated to A (resp. B), where $\alpha + \beta = 1$ and $0 \leq \alpha, \beta \leq 1$. Agents A's utility function is $u_A(\alpha) = \alpha$ and that of agent B is $u_B(\beta) = \beta$.

(i) What is the set of Pareto optimal allocations in this economy?

(ii) Suppose A is asked to cut the cake in two parts, after which B can choose which of the two segments to pick for herself, leaving the other segment for agent A. How should A cut the cake?

(iii) Suppose A is altruistic, and his utility function puts weight on what B obtains, i.e. $u_A(\alpha, \beta) = \alpha + \mu\beta$, where μ is the weight on agent B's utility. (a) If $0 < \mu < 1$, does the answer to either 8(i) or 8(ii) change? (b) What if $\mu > 1$? [5]+[5]+[10]

9. A firm uses four inputs to produce an output with a production function

$$f(x_1, x_2, x_3, x_4) = \min\{x_1, x_2\} + \min\{x_3, x_4\}.$$

(i) Suppose that 1 unit of output is to be produced and factor prices are $1, 2, 3$ and 4 for x_1, x_2, x_3 and x_4 respectively. Solve for the optimal factor demands.

(ii) Derive the cost function.

(iii) What kind of returns to scale does this technology exhibit? [6]+[8]+[6]

10. Consider an IS-LM model where the sectoral demand functions are given by

$$C = 90 + 0.75Y,$$
$$G = 30, I = 300 - 50r,$$
$$(\frac{M}{P})_d = 0.25Y - 62.5r, (\frac{M}{P})_s = 500.$$

Any disequilibrium in the international money market is corrected instantaneously through a change in r. However, any disequilibrium in the goods market, which is corrected through a change in Y, takes much longer to be eliminated.

(a) Now consider an initial situation where $Y = 2500, r = 1/5$. What is the change in the level of I that must occur before there is any change in the level of Y?

(b) Draw a graph to explain your answer.

(c) Calculate the value of (r, Y) that puts both the money and goods market in equilibrium. What is the value of investment at this point compared to $(r = 2, Y = 2500)$?

[10]+[5]+[5]