Test code: ME I/ME II, 2006

Syllabus for ME I, 2006

Matrix Algebra: Matrices and Vectors, Matrix Operations.

Permutation and Combination.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables, Unconstrained optimization, Definite and Indefinite Integrals: integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables**.**

Linear Programming: Formulations, statements of Primal and Dual problems, Graphical solutions.

Theory of Polynomial Equations (up to third degree).

Elementary Statistics: Measures of central tendency; dispersion, correlation, Elementary probability theory, Probability mass function, Probability density function and Distribution function.

Sample Questions for ME I (Mathematics), 2006

For each of the following questions four alternative answers are provided. Choose the answer that you consider to be the most appropriate for a question.

1. If $f(x) = \log\left(\frac{1+x}{1-x}\right), \quad 0 < x < 1$, then $f\left(\frac{2x}{1+x^2}\right)$ equals

(A) $2f(x)$; (B) $\frac{f(x)}{2}$; (C) $(f(x))^2$; (D) none of these.

2. If $u = \phi(x-y, y-z, z-x)$, then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ equals

(A) 0; (B) 1; (C) u; (D) none of these.

3. Let A and B be disjoint sets containing m and n elements, respectively, and let $C = A \bigcup B$. The number of subsets S of C that contain k elements and that also have the property that $S \bigcap A$ contains i elements is

(A) $\binom{m}{i}$; (B) $\binom{n}{i}$; (C) $\binom{m}{k-i}\binom{n}{i}$; (D) $\binom{m}{i}\binom{n}{k-i}$.

4. The number of disjoint intervals over which the function $f(x) = \left|0.5x^2 - |x|\right|$ is decreasing is

(A) one; (B) two; (C) three; (D) none of these.

5. For a set of real numbers $x_1, x_2, ..., x_n$, the root mean square (RMS) defined as RMS = $\left\{\frac{1}{N}\sum_{i=1}^{n} x_i^2\right\}^{1/2}$ is a measure of central tendency. If AM denotes the arithmetic mean of the set of numbers, then which of the following statements is correct?

(A) RMS < AM always; (B) RMS > AM always;
(C) RMS < AM when the numbers are not all equal;
(D) RMS > AM when numbers are not all equal.

6. Let *f(x)* be a function of real variable and let Δf be the function $\Delta f\,(\mathrm{x}) = f(x+1) - f(x)$. For $\mathrm{k} > 1$, put $\Delta^k f = \Delta(\Delta^{k-1} f)$. Then $\Delta^k f(x)$ equals

(A) $\sum_{j=0}^{k}(-1)^j\binom{k}{j}f(x+j)$; (B) $\sum_{j=0}^{k}(-1)^{j+1}\binom{k}{j}f(x+j)$;
(C) $\sum_{j=0}^{k}(-1)^j\binom{k}{j}f(x+k-j)$; (D) $\sum_{j=0}^{k}(-1)^{j+1}\binom{k}{j}f(x+k-j)$.

7. Let $I_n = \int_0^\infty x^n e^{-x} dx$, where n is some positive integer. Then I_n equals

(A) $n! - nI_{n-1;}$ (B) $n! + nI_{n-1;}$ (C) $nI_{n-1;}$ (D) none of these.

8. If $x^3 = 1$, then

$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ equals

(A) $(cx^2 + bx + a)\begin{vmatrix} 1 & b & c \\ x & c & a \\ x^2 & a & b \end{vmatrix}$; (B) $(cx^2 + bx + a)\begin{vmatrix} x & b & c \\ 1 & c & a \\ x^2 & a & b \end{vmatrix}$;

(C) $(cx^2 + bx + a)\begin{vmatrix} x^2 & b & c \\ x & c & a \\ 1 & a & b \end{vmatrix}$; (D) $(cx^2 + bx + a)\begin{vmatrix} 1 & b & c \\ x^2 & c & a \\ x & a & b \end{vmatrix}$.

9. Consider any integer $I = m^2 + n^2$, where m and n are any two odd integers. Then

(A) I is never divisible by 2;
(B) I is never divisible by 4;
(C) I is never divisible by 6;
(D) none of these.

10. A box has 10 red balls and 5 black balls. A ball is selected from the box. If the ball is red, it is returned to the box. If the ball is black, it and 2 additional black balls are added to the box. The probability that a second ball selected from the box will be red is

(A) $\frac{47}{72}$; (B) $\frac{25}{72}$; (C) $\frac{55}{153}$; (D) $\frac{98}{153}$.

11. Let $f(x) = \dfrac{\log\left(1+\dfrac{x}{p}\right) - \log\left(1-\dfrac{x}{q}\right)}{x}$, $x \neq 0$. If f is continuous at $x = 0$, then the value of $f(0)$ is

(A) $\dfrac{1}{p} - \dfrac{1}{q}$; (B) $p+q$; (C) $\dfrac{1}{p} + \dfrac{1}{q}$; (D) none of these.

12. Consider four positive numbers x_1, x_2, y_1, y_2 such that $y_1\, y_2 > x_1\, x_2$. Consider the number $S = (x_1 y_2 + x_2 y_1) - 2x_1 x_2$. The number S is

(A) always a negative integer;
(B) can be a negative fraction;
(C) always a positive number;
(D) none of these.

13. Given $x \geq y \geq z$, and $x + y + z = 12$, the maximum value of $x + 3y + 5z$ is

(A) 36; (B) 42; (C) 38; (D) 32.

14. The number of positive pairs of integral values of (x, y) that solves $2xy - 4x^2 + 12x - 5y = 11$ is

(A) 4; (B) 1; (C) 2; (D) none of these.

15. Consider any continuous function f: [0, 1] $\rightarrow$ [0, 1]. Which one of the following statements is ***incorrect***?

(A) f always has at least one maximum in the interval [0, 1];
(B) f always has at least one minimum in the interval [0, 1];
(C) $\exists$ x $\in$ [0, 1] such that $f(x) = x$;
(D) the function f must always have the property that $f(0) \in \{0, 1\}$, $f(1) \in \{0, 1\}$ and $f(0) + f(1) = 1$.

Syllabus for ME II (Economics), 2006

Microeconomics: Theory of consumer behaviour, Theory of production, Market forms (Perfect competition, Monopoly, Price Discrimination, Duopoly – Cournot and Bertrand (elementary problems)) and Welfare economics.

Macroeconomics: National income accounting, Simple model of income determination and Multiplier, IS – LM model (with comparative statics), Harrod – Domar and Solow models, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2006

1.(a) There are two sectors producing the same commodity. Labour is perfectly mobile between these two sectors. Labour market is competitive and the representative firm in each of the two sectors maximizes profit. If there are 100 units of labour and the production function for sector i is: $F(L_i) = 15\sqrt{L_i}$, $i = 1,2$, find the allocation of labour between the two sectors.

(b) Suppose that prices of all variable factors and output double. What will be its effect on the short-run equilibrium output of a competitive firm? Examine whether the short-run profit of the firm will double.

(c) Suppose in year 1 economic activities in a country constitute only production of wheat worth Rs. 750. Of this, wheat worth Rs. 150 is exported and the rest remains unsold. Suppose further that in year 2 no production takes place, but the unsold wheat of year 1 is sold domestically and residents of the country import shirts worth Rs. 250. Fill in, with adequate explanation, the following chart :

Year	GDP	= Consumption	+ Investment	+ Export	- Import
1	____	____	____	____	____
2	____	____	____	____	____

2. A price-taking farmer produces a crop with labour L as the only input. His production function is: $F(L) = 10\sqrt{L} - 2L$. He has 4 units of labour

in his family and he cannot hire labour from the wage labour market. He does not face any cost of employing family labour.

(a) Find out his equilibrium level of output.

(b) Suppose that the government imposes an income tax at the rate of 10 per cent. How does this affect his equilibrium output?

(c) Suppose an alternative production technology given by: $F(L) = 11\sqrt{L} - L - 15$ is available. Will the farmer adopt this alternative technology? Briefly justify your answer.

3. Suppose a monopolist faces two types of consumers. In type *I* there is only one person whose demand for the product is given by : $Q_1 = 100 - P$, where *P* represents price of the good. In type *II* there are *n* persons, each of whom has a demand for one unit of the good and each of them wants to pay a maximum of Rs. 5 for one unit. Monopolist cannot price discriminate between the two types. Assume that the cost of production for the good is zero. Does the equilibrium price depend on n ? Give reasons for your answer.

4. The utility function of a consumer is: $U(x, y) = xy$. Suppose income of the consumer (M) is 100 and the initial prices are $P_x = 5$, $P_y = 10$. Now suppose that P_x goes up to 10, P_y and *M* remaining unchanged. Assuming Slutsky compensation scheme, estimate price effect, income effect and substitution effect.

5. Consider an *IS-LM* model for a closed economy. Private consumption depends on disposable income. Income taxes (*T*) are lump-sum. Both private investment and speculative demand for money vary inversely with interest rate (*r*). However, transaction demand for money depends not on income (*y*) but on disposable income (y_d). Argue how the equilibrium values of private investment, private saving, government saving, disposable income and income will change, if the government raises *T*.

6. An individual enjoys bus ride. However, buses emit smoke which he dislikes. The individual's utility function is: $U = U(x, s)$, where *x* is the distance (in km) traveled by bus and *s* is the amount of smoke consumed from bus travel.

(a) What could be the plausible alternative shapes of indifference curve between x and s?

(b) Suppose, smoke consumed from bus travel is proportional to the distance traveled: $s = \alpha x$ (α is a positive parameter). Suppose further that the bus fare per km is p and that the individual has money income M to spend on bus travel. Show the budget set of the consumer in an (s, x) diagram.

(c) What can you say about an optimal choice of the individual? Will he necessarily exhaust his entire income on bus travel?

7. (a) Suppose the labour supply (l) of a household is governed by maximization of its utility (u): $u = c^{2/3}h^{1/3}$, where c is the household's consumption and h is leisure enjoyed by the household (with $h + l = 24$). Real wage rate (w) is given and the household consumes the entire labour income (wl). What is the household's labour supply? Does it depend on w?

(b) Consider now a typical Keynesian (closed) economy producing a single good and having a single household. There are two types of final expenditure – viz., investment autonomously given at 36 units and household consumption (c) equalling the household's labour income (wl). It is given that $w = 4$. Firms produce aggregate output (y) according to the production function: $y = 24\sqrt{l}$. Find the equilibrium level of output and employment. Is there any involuntary unemployment? If so, how much?

8. Suppose an economic agent's life is divided into two periods, the first period constitutes her youth and the second her old age. There is a single consumption good, C, available in both periods and the agent's utility function is given by

$$u(C_1, C_2) = \frac{C_1^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho}\,\frac{C_2^{1-\theta} - 1}{1-\theta}, \qquad 0 < \theta < 1, \rho > 0,$$

where the first term represents utility from consumption during youth. The second term represents discounted utility from consumption in old age, $1/(1+\rho)$ being the discount factor. During the period, the agent has a

unit of labour which she supplies inelastically for a wage rate w. Any savings (i.e., income minus consumption during the first period) earns a rate of interest r, the proceeds from which are available in old age in units of the only consumption good available in the economy. Denote savings by s. The agent maximizes utility subjects to her budget constraint.

i) Show that θ represents the elasticity of marginal utility with respect to consumption in each period.

ii) Write down the agent's optimization problem, i.e., her problem of maximizing utility subject to the budget constraint.

iii) Find an expression for s as a function of w and r.

iv) How does s change in response to a change in r? In particular, show that this change depends on whether θ exceeds or falls short of unity.

v) Give an intuitive explanation of your finding in (iv)

9. A consumer consumes only two commodities x_1 and x_2. Suppose that her utility function is given by $U(x_1, x_2) = \min (2x_1, x_2)$.

(i) Draw a representative indifference curve of the consumer.

(ii) Suppose the prices of the commodities are Rs.5 and Rs.10 respectively while the consumer's income is Rs. 100. What commodity bundle will the consumer purchase?

(iii) Suppose the price of commodity 1 now increases to Rs. 8. Decompose the change in the amount of commodity 1 purchased into income and substitution effects.

10. A price taking firm makes machine tools Y using labour and capital according to the production function $Y = K^{0.25} L^{0.25}$. Labour can be hired at the beginning of every week while capital can be hired only at the beginning of every month. Let one month be considered as long run period and one week as short run period. Further assume that one month equals four weeks. The wage rate per week and the rental rate of capital per month are both 10.

(i) Given the above information, find the short run and the long run cost functions of the firm.

(ii) At the beginning of the month of January, the firm is making long run decisions given that the price of machine tools is 400. What is the long run profit maximizing number of machine tools? How many units of labour and capital should the firm hire at the beginning of January?

11. Consider a neo-classical one-sector growth model with the production function $Y = \sqrt{KL}$. If 30% of income is invested and capital stock depreciates at the rate of 7% and labour force grows at the rate of 3%, find out the level of per capita income in the steady-state equilibrium.