

Test code: ME I/ME II, 2008

Syllabus for ME I, 2008

Matrix Algebra: Matrices and Vectors, Matrix Operations.

Permutation and Combination.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables, Unconstrained optimization, Definite and Indefinite Integrals: integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Linear Programming: Formulations, statements of Primal and Dual problems, Graphical solutions.

Theory of Polynomial Equations (up to third degree).

Elementary Statistics: Measures of central tendency; dispersion, correlation, Elementary probability theory, Probability mass function, Probability density function and Distribution function.

Sample Questions for ME I (Mathematics), 2008

1. $\int \frac{dx}{x + x \log x}$ equals
- $\log|x + x \log x| + \text{constant}$
 - $\log|1 + x \log x| + \text{constant}$
 - $\log|\log x| + \text{constant}$
 - $\log|1 + \log x| + \text{constant}$.
2. The inverse of the function $\sqrt{-1+x}$ is
- $\frac{1}{\sqrt{x-1}}$, (b) $x^2 + 1$, (c) $\sqrt{x-1}$, (d) none of these.
3. The domain of continuity of the function $f(x) = \sqrt{x} + \frac{x+1}{x-1} - \frac{x+1}{x^2+1}$ is
- $[0,1)$, (b) $(1,\infty)$, (c) $[0,1) \cup (1,\infty)$, (d) none of these
4. Consider the following linear programme:
- minimise $x - 2y$
- subject to $x + 3y \geq 3$
- $3x + y \geq 3$
- $x + y \leq 3$
- An optimal solution of the above programme is given by
- $x = \frac{3}{4}, y = \frac{3}{4}$.
 - $x = 0, y = 3$
 - $x = -1, y = 3$.
 - none of (a), (b) and (c).
5. Consider two functions $f_1 : \{a_1, a_2, a_3\} \rightarrow \{b_1, b_2, b_3, b_4\}$ and $f_2 : \{b_1, b_2, b_3, b_4\} \rightarrow \{c_1, c_2, c_3\}$. The function f_1 is defined by $f_1(a_1) = b_1, f_1(a_2) = b_2, f_1(a_3) = b_3$ and the function f_2 is defined by $f_2(b_1) = c_1, f_2(b_2) = c_2, f_2(b_3) = c_2, f_2(b_4) = c_3$. Then the mapping $f_2 \circ f_1 : \{a_1, a_2, a_3\} \rightarrow \{c_1, c_2, c_3\}$ is
- a composite and one – to – one function but not an onto function.
 - a composite and onto function but not a one – to – one function.
 - a composite, one – to – one and onto function.
 - not a function.

6. If $x = t^{\frac{1}{t-1}}$ and $y = t^{\frac{t}{t-1}}$, $t > 0$, $t \neq 1$ then the relation between x and y is
 (a) $y^x = x^y$, (b) $x^y = y^x$, (c) $x^y = y^x$, (d) $x^y = y^{\frac{1}{x}}$.
7. The maximum value of $T = 2x_B + 3x_S$ subject to the constraint $20x_B + 15x_S \leq 900$, where $x_B \geq 0$ and $x_S \geq 0$, is
 (a) 150, (b) 180, (c) 200, (d) none of these.
8. The value of $\int_0^2 [x]^n f'(x) dx$, where $[x]$ stands for the integral part of x , n is a positive integer and f' is the derivative of the function f , is
 (a) $(n + 2^n)(f(2) - f(0))$, (b) $(1 + 2^n)(f(2) - f(1))$,
 (c) $2^n f(2) - (2^n - 1)f(1) - f(0)$, (d) none of these.
9. A surveyor found that in a society of 10,000 adult literates 21% completed college education, 42% completed university education and remaining 37% completed only school education. Of those who went to college 61% reads newspapers regularly, 35% of those who went to the university and 70% of those who completed only school education are regular readers of newspapers. Then the percentage of those who read newspapers regularly completed only school education is
 (a) 40%, (b) 52%, (c) 35%, (d) none of these.
10. The function $f(x) = x|x|e^{-x}$ defined on the real line is
 (a) continuous but not differentiable at zero,
 (b) differentiable only at zero,
 (c) differentiable everywhere,
 (d) differentiable only at finitely many points.
11. Let X be the set of positive integers denoting the number of tries it takes the Indian cricket team to win the World Cup. The team has equal odds for winning or losing any match. What is the probability that they will win in odd number of matches?
 (a) $1/4$, (b) $1/2$, (c) $2/3$, (d) $3/4$.

12. Three persons X, Y, Z were asked to find the mean of 5000 numbers, of which 500 are unities. Each one did his own simplification.

X's method: Divide the set of number into 5 equal parts, calculate the mean for each part and then take the mean of these.

Y's method: Divide the set into 2000 and 3000 numbers and follow the procedure of A.

Z's method: Calculate the mean of 4500 numbers (which are $\neq 1$) and then add 1.
Then

- (a) all methods are correct,
- (b) X's method is correct, but Y and Z's methods are wrong,
- (c) X's and Y's methods are correct but Z's methods is wrong,
- (d) none is correct.

13. The number of ways in which six letters can be placed in six directed envelopes such that exactly four letters are placed in correct envelopes and exactly two letters are placed in wrong envelopes is

- (a) 1, (b) 15, (c) 135. (d) None of these.

14. The set of all values of x for which the inequality $|x - 3| + |x + 2| < 11$ holds is

- (a) $(-3, 2)$, (b) $(-5, 2)$, (c) $(-5, 6)$, (d) none of these.

15. The function $f(x) = x^4 - 4x^3 + 16x$ has

- (a) a unique maximum but no minimum,
- (b) a unique minimum but no maximum,
- (c) a unique maximum and a unique minimum,
- (d) neither a maximum nor a minimum.

16. Consider the number $K(n) = (n+3)(n^2 + 6n + 8)$ defined for integers n . Which of the following statements is correct?

- (a) $K(n)$ is always divisible by 4,
- (b) $K(n)$ is always divisible by 5,
- (c) $K(n)$ is always divisible by 6,
- (d) Statements (a), (b) and (c) are incorrect.

17. 25 books are placed at random on a shelf. The probability that a particular pair of books shall be always together is

- (a) $\frac{2}{25}$, (b) $\frac{1}{25}$, (c) $\frac{1}{300}$, (d) $\frac{1}{600}$.

18. $P(x)$ is a quadratic polynomial such that $P(1) = -P(2)$. If -1 is a root of the equation, the other root is

- (a) $\frac{4}{5}$, (b) $\frac{8}{5}$, (c) $\frac{6}{5}$, (d) $\frac{3}{5}$.

19. The correlation coefficients between two variables X and Y obtained from the two equations $2x + 3y - 1 = 0$ and $5x - 2y + 3 = 0$ are

- (a) equal but have opposite signs,
(b) $-\frac{2}{3}$ and $\frac{2}{5}$,
(c) $\frac{1}{2}$ and $-\frac{3}{5}$,
(d) Cannot say.

20. If a, b, c, d are positive real numbers then $\frac{a}{b} + \frac{b}{c} + \frac{c}{d} + \frac{d}{a}$ is always

- (a) less than $\sqrt{2}$,
(b) less than 2 but greater than or equal to $\sqrt{2}$,
(c) less than 4 but greater than or equal to 2,
(d) greater than or equal to 4.

21. The range of value of x for which the inequality $\log_{(2-x)}(x-3) \geq -1$ holds is

- (a) $2 < x < 3$, (b) $x > 3$, (c) $x < 2$, (d) no such x exists.

22. The equation $5x^3 - 5x^2 + 2x - 1$ has

- (a) all roots between 1 and 2,
(b) all negative roots,
(c) a root between 0 and 1,
(d) all roots greater than 2.

23. The probability density of a random variable is

$$f(x) = ax^2 \exp^{-kx} \quad (k > 0, 0 \leq x \leq \infty)$$

Then, a equals

- (a) $\frac{k^3}{2}$, (b) $\frac{k}{2}$, (c) $\frac{k^2}{2}$, (d) k .

24. Let $x = r$ be the mode of the distribution with probability mass function

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}. \text{ Then which of the following inequalities hold.}$$

- (a) $(n+1)p - 1 < r < (n+1)p$,
(b) $r < (n+1)p - 1$,
(c) $r > (n+1)p$,
(d) $r < np$.

25. Let $y = (y_1, \dots, y_n)$ be a set of n observations with $y_1 \leq y_2 \leq \dots \leq y_n$. Let $y' = (y_1, y_2, \dots, y_j + \delta, \dots, y_k - \delta, \dots, y_n)$ where $y_k - \delta > y_{k-1} > \dots > y_{j+1} > y_j + \delta$, $\delta > 0$. Let σ : standard deviation of y and σ' : standard deviation of y' . Then
 (a) $\sigma < \sigma'$, (b) $\sigma' < \sigma$, (c) $\sigma' = \sigma$, (d) nothing can be said.

26. Let x be a r.v. with pdf $f(x)$ and let $F(x)$ be the distribution function. Let

$$r(x) = \frac{xf(x)}{1-F(x)}. \text{ Then for } x < e^\mu \text{ and } f(x) = \frac{e^{-\frac{(\log x - \mu)^2}{2}}}{x\sqrt{2\pi}}, \text{ the function } r(x) \text{ is}$$

- (a) increasing in x ,
 (b) decreasing in x ,
 (c) constant,
 (d) none of the above.
27. A square matrix of order n is said to be a bistochastic matrix if all of its entries are non-negative and each of its rows and columns sum to 1. Let $y_{n \times 1} = P_{n \times n} \cdot x_{n \times 1}$ where elements of y are some rearrangements of the elements of x . Then
 (a) P is bistochastic with diagonal elements 1,
 (b) P cannot be bistochastic,
 (c) P is bistochastic with elements 0 and 1,
 (d) P is a unit matrix.

28. Let $f_1(x) = \frac{x}{x+1}$. Define $f_n(x) = f_1(f_{n-1}(x))$, where $n \geq 2$. Then $f_n(x)$ is
 (a) decreasing in n , (b) increasing in n , (c) initially decreasing in n and then increasing in n , (d) initially increasing in n and then decreasing n .

29. $\lim_{n \rightarrow \infty} \frac{1 - x^{-2n}}{1 + x^{-2n}}, x > 0$ equals

- (a) 1, (b) -1, (c) 0. (d) The limit does not exist.

30. Consider the function $f(x_1, x_2) = \max\{6 - x_1, 7 - x_2\}$. The solution (x_1^*, x_2^*) to the optimization problem minimize $f(x_1, x_2)$ subject to $x_1 + x_2 = 21$ is

- (a) $(x_1^* = 10.5, x_2^* = 10.5)$,
 (b) $(x_1^* = 11, x_2^* = 10)$,
 (c) $(x_1^* = 10, x_2^* = 11)$,
 (d) None of these.

Syllabus for ME II (Economics), 2008

Microeconomics: Theory of consumer behaviour, Theory of Production, Market Structures under Perfect Competition, Monopoly, Price Discrimination, Duopoly with Cournot and Bertrand Competition (elementary problems) and Welfare economics.

Macroeconomics: National Income Accounting, Simple Keynesian Model of Income Determination and the Multiplier, IS-LM Model, Model of Aggregate Demand and Aggregate Supply, Harrod-Domar and Solow Models of Growth, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2008

1. There are two individuals A and B and two goods X and Y. The utility functions of A and B are given by $U_A = X_A$ and $U_B = X_B^2 + Y_B^2$ respectively where X_i, Y_i are consumption levels of the two goods by individual $i, i = A, B$.

- Draw the indifference curves of A and B.
- Suppose A is endowed with 10 units of Y and B with 10 units of X. Indicate the endowment point in a box diagram.
- Draw the set of Pareto optimal allocation points in the box diagram.

2. Suppose an economy's aggregate output (Y) is given by the following production function:

$$Y = U N^\alpha, (0 < \alpha < 1)$$

where U , a random variable, represents supply shock. Employment of labour (N) is determined by equating its marginal product to $\frac{W}{P}$, where W is nominal wage and P is price level.

Use the notations: $u = \log \alpha + \frac{1}{\alpha} \log U$; $p = \log P$; $w = \log W$ and $y = \log Y$.

- Obtain the aggregate supply function (y) in terms of p , w , and u .
- Add the following relations:

Wages are indexed: $w = \theta p$, ($0 \leq \theta \leq 1$)

Aggregate demand: $y = m - p$, ($m =$ logarithm of money, a policy variable)

Find the solution of y in terms of m and u .

- Does monetary policy affect output (i) if indexation is partial ($0 < \theta < 1$), (ii) indexation is full ($\theta = 1$)?
- Does the real shock affect output more when indexation is higher? Explain.

3. Two firms 1 and 2 sell a single, homogeneous, infinitely divisible good in a market. Firm 1 has 40 units to sell and firm 2 has 80 units to sell. Neither firm can produce any more units. There is a demand curve: $p = a - q$, where q is the total amount placed by the firms in the market. So if q_i is the amount placed by firm i , $q = q_1 + q_2$ and p is the price that emerges. a is positive and a measure of market size. It is known that a is either 100 or 200. The value of a is observed by both firms. After they observe the value of a , each firm decides whether or not to destroy a part of its output. This decision is made simultaneously and independently by the firms. Each firm faces a constant per unit cost of destruction

equal to 10. Whatever number of units is left over after destruction is sold by the firm in the market.

Show that a firm's choice about the amount it wishes to destroy is independent of the amount chosen by the other firm. Show also that the amount destroyed by firm 2 is always positive, while firm 1 destroys a part of its output if and only if $a = 100$.

4. (a) Two commodities, X and Y , are produced with identical technology and are sold in competitive markets. One unit of labour can produce one unit of each of the two commodities. Labour is the only factor of production; and labour is perfectly mobile between the two sectors. The representative consumer has the utility function: $U = \sqrt{XY}$; and his income is Rs. 100/-. If 10 units of labour are available, find out the equilibrium wage in the competitive labour market.

(b) Consider an economy producing a single good by a production function

$$Y = \min \{K, L\}$$

where Y is the output of the final good. K and L are input use of capital and labour respectively. Suppose this economy is endowed with 100 units of capital and labour supply L_s is given by the function

$$L_s = 50w,$$

where w is the wage rate.

Assuming that all markets are competitive find the equilibrium wage and rental rate.

5. The following symbols are used: Y = output, N = employment, W = nominal wage, P = price level, P^e = expected price level.

The Lucas supply function is usually written as:

$$\log Y = \log Y^* + \lambda (\log P - \log P^e)$$

where Y^* is the natural level of output. Consider an economy in which labour supply depends positively on the expected real wage:

$$\frac{W}{P^e} = N^\sigma, (\sigma > 0) \quad (\text{labour supply})$$

Firms demand labour up to the point where its marginal product equals the given (actual) real wage $\left(\frac{W}{P}\right)$ and firm's production function is:

$$Y = N^\alpha, (0 < \alpha < 1)$$

- (a) Find the labour demand function.
- (b) Equate labour demand with labour supply to eliminate W . You will get an expression involving P , P^e and N . Derive the Lucas supply function in the form given above and find the expressions for λ and Y^* .
- (c) How is this type of model referred to in the literature? Explain

6. Consider an IS – LM model given by the following equations

$$C = 200 + .5 Y_D$$

$$I = 150 - 1000 r$$

$$T = 200$$

$$G = 250$$

$$\left(\frac{M}{P}\right)^d = 2Y - 4000i$$

$$\left(\frac{M}{P}\right)^s = 1600$$

$$i = r - \Pi^e$$

where C is consumption, Y_D is disposable income, I is investment, r is real rate of interest, i is nominal rate of interest, T is tax, G is government expenditure,

$\left(\frac{M}{P}\right)^d$ and $\left(\frac{M}{P}\right)^s$ are real money demand and real money supply respectively and

and Π^e is the expected rate of inflation. The current price level P remains always rigid.

- (a) Assuming that $\Pi^e = 0$, i.e., the price level is expected to remain unchanged in future, determine the equilibrium levels of income and the rates of interest.
- (b) Suppose there is a *temporary* increase in nominal money supply by 2%. Find the new equilibrium income and the rates of interest.
- (c) Now assume that the 2% increase in nominal money supply is *permanent* leading to a 2% increase in the expected future price level. Work out the new equilibrium income and the rates of interest.

7. A firm is contemplating to hire a salesman who would be entrusted with the task of selling a washing machine. The hired salesman is efficient with probability 0.25 and inefficient with probability 0.75 and there is no way to tell, by looking at the salesman, if he is efficient or not. An efficient salesman can sell the washing machine with probability 0.8 and an inefficient salesman can sell the machine with probability 0.4. The firm makes

a profit of Rs. 1000 if the machine is sold and gets nothing if it is not sold. In either case, however, the salesman has to be paid a wage of Rs. 100.

- (a) Calculate the expected profit of the firm.
(b) Suppose instead of a fixed payment, the firm pays a commission of t % on its profit to the salesman (i.e., if the good is sold the salesman gets Rs. $1000 \times \frac{t}{100}$ and nothing if the good remains unsold). A salesman, irrespective of whether he is efficient or inefficient, has an alternative option of working for Rs. 80. A salesman knows whether he is efficient or not and cares only about the expected value of his income: find the value of t that will maximize the expected profit of the firm.

8. (a) On a tropical island there are 100 boat builders, numbered 1 through 100. Each builder can build up to 12 boats a year and each builder maximizes profit given the market price. Let y denote the number of boats built per year by a particular builder, and for each i , from 1 to 100, boat builder has a cost function $C_i(y) = 11 + iy$. Assume that in the cost function the fixed cost, 11, is a quasi-fixed cost, that is, it is only paid if the firm produces a positive level of output. If the price of a boat is 25, how many builders will choose to produce a positive amount of output and how many boats will be built per year in total?

(b) Consider the market for a particular good. There are two types of customers: those of type 1 are the low demand customers, each with a demand function of the form $p = 10 - q_1$, and those of type 2, who are the high demand customers, each with a demand function of the form $p = 2(10 - q_2)$. The firm producing the product is a monopolist in this market and has a cost function $C(q) = 4q^2$ where $q = q_1 + q_2$.

- (i) Suppose the firm is unable to prevent the customers from selling the good to one another, so that the monopolist cannot charge different customers different prices. What prices per unit will the monopolist charge to maximize its total profit and what will be the equilibrium quantities to be supplied to the two groups in equilibrium?
- (ii) Suppose the firm realizes that by asking for IDs it can identify the types of the customers (for instance, type 1's are students who can be identified using their student IDs). It can thus charge different per unit prices to the two groups, if it is optimal to do so. Find the profit maximizing prices to be charged to the two groups.

9. Consider the following box with 16 squares:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

There are two players 1 and 2, and the game begins with player 1 selecting one of the boxes marked 1 to 16. Following such a selection, the selected box, as well as all boxes in the square of which the selected box constitutes the *leftmost and lowest* corner, will be deleted. For example, if he selects box 7, then all the boxes, 3, 4, 7 and 8 are deleted. Similarly, if he selects box 9, then all boxes 1 to 12 are deleted. Next it is player 2's turn to select a box from the remaining boxes. The same deletion rule applies in this case. It is then player 1's turn again, and so on. Whoever deletes the last box loses the game? What is a winning strategy for player 1 in this game?

10. (i) Mr. A's yearly budget for his car is Rs. 100,000, which he spends completely on petrol (P) and on all other expenses for his car (M). All other expenses for car (M) is measured in Rupees, so you can consider that price of M is Re. 1. When price of petrol is Rs. 40 per liter, Mr. A buys 1,000 liters per year.

The petrol price rises to Rs. 50 per liter, and to offset the harm to Mr. A, the government gives him a cash transfer of Rs. 10,000 per year.

- Write down Mr. A's yearly budget equation under the 'price rise plus transfer' situation.
- What will happen to his petrol consumption – increase, decrease, or remain the same?
- Will he be better or worse off after the price rise plus transfer than he was before?

[Do not refer to any utility function or indifference curves to answer (b) and (c)]

(iii) Mr. B earns Rs. 500 today and Rs. 500 tomorrow. He can save for future by investing today in bonds that return tomorrow the principal plus the interest. He can also borrow from his bank paying an interest. When the interest rates on both bank loans and bonds are 15% Mr. B chooses neither to save nor to borrow.

- Suppose the interest rate on bank loans goes up to 30% and the interest rate on bonds fall to 5%. Write down the equation of the new budget constraint and draw his budget line.
- Will he lend or borrow? By how much?