M.Sc. Economics

Indian Institute of Technology Delhi

Entrance Examination 2023

Set A

Instructions: Read Carefully

- There are two sections. Section I has 20 one-mark questions. For each, a correct answer fetches one mark, an incorrect answer penalizes negative $\frac{1}{3}$, and no answer gives no marks. Section II has 40 questions, with a correct answer getting two marks, an incorrect answer getting negative $\frac{2}{3}$, and no answer to a question getting zero.
- Maximum possible marks is 100.
- You are allowed to use a non-programmable calculator. Cheat sheets, mobile phones, programmable calculators, and any other electronic devices are strictly not allowed.
- You will need to submit the entire question paper (including the rough sheets) along with the OMR sheet.
- You should use a black/blue ballpoint pen only to write particulars on the test booklet and darken the correct response on the OMR sheet. See the OMR sheet for instructions on how to fill a bubble correctly.
- There should be 32 pages in the exam booklet. Check your copy and request a new booklet if you do not have 32 pages.
- If any candidate is caught using unfair means during the exam, he/she will be immediately disqualified and will be asked to leave the exam hall.

I Multiple Choice Questions (One mark each)

1. In the standard IS-LM model, in which government spending is exogenous, investment is a function of interest rate, and consumption is a function of income and interest rate, an increase in Government spending (G) without changing taxes

   A. necessarily has a positive effect on equilibrium consumption.

   B. necessarily has a negative effect on equilibrium consumption.

   C. has an ambiguous effect on equilibrium consumption.

   D. necessarily has a neutral effect on equilibrium consumption.

2. According to the standard Solow growth model, which of the following statements is FALSE?

   A. A country that experiences higher population growth than another will have a lower output per worker in a steady-state.

   B. Steady-state consumption in a country that saves more will always be higher than the steady-state consumption of a country with a lower savings rate.

   TRUE C. Capital accumulation alone can not sustain long-run growth in capital per worker.

   D. Consumption per capita in the golden rule steady state cannot be less compared to other steady states.

3. Consider the following utility function:

$$u(x_1, x_2) = \min(2x_1 + x_2, x_1 + 2x_2)$$

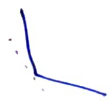
For what value of $\frac{p_1}{p_2}$ will the unique optimum be $x_1^* = 0$ and $x_2^* = \frac{m}{p_2}$ where p_1, p_2 and m denote the prices of x_1, x_2, and income, respectively.

   A. $\frac{p_1}{p_2} \leq 2$.

   B. $\frac{p_1}{p_2} > 2$.

   C. $\frac{p_1}{p_2} < \frac{1}{2}$.

   D. $\frac{p_1}{p_2} \geq \frac{1}{2}$.



4. Consider the following utility function: $u(x_1, x_2) = \min(x_1, 2x_2)$ and the budget line is negatively sloped. Suppose we increase the price of good 1, then the effect of change of this price on Hicksian demand is

   A. positive.

   B. negative.

   C. zero.

   D. ambiguous.

$x_1 = 2x_2$

$p_1x_1 + p_2x_2 = m$

$\frac{m}{(2p_1+p_2)} = x_2$

$\frac{m}{(p_1+\frac{p_2}{2})} = x_1$

$x_1 \Rightarrow \frac{2m}{2p_1+p_2} = x_1$

$u = \frac{2m}{2p_1+p_2}$

$\frac{v(2p_1+p_1)}{2} = e$

$\frac{\partial e}{\partial p_1} = \frac{v}{2}(2) = v = \bar{u} > 0$ $= 0$

5. For a quasilinear utility with two commodities X, Y: $u(x, y) = \phi(x) + y$, where $\phi' > 0$ and $\phi'' < 0$. For a given budget, in an interior equilibrium,

A. the income effect on X is always zero.

B. the income effect on Y is always zero.

C. the substitution effect on X is always zero.

D. none of the above is correct.

6. A cake of size one is to be divided between two individuals, 1 and 2. Let x_i be the share of the cake for individual $i, i = 1, 2$, where $0 \leq x_i \leq 1$. The utility functions are $u_1(x_1, x_2) = x_1$, and $u_2(x_1, x_2) = x_2 + |x_1 - x_2|$, where for any real a, define $|a|$ as the absolute value of a. The Pareto optimal cake divisions include:

   A. $(1, 0)$.

   B. $(1/2, 1/2)$.

   C. $(3/4, 1/4)$.

   D. None of the above

7. A monopoly firm sells its product in Delhi, where the elasticity of demand is -2, and in Goa, where the elasticity of demand is -5. Its marginal cost of production is Rs. 10. At what price does the monopolist sell the product in each state if resale is impossible? (Prices in Delhi and Goa are denoted by p_D and p_G, respectively.)

   C A. $p_D = 20$ and $p_G = 12.5$.

   B. $p_D = 12.5$ and $p_G = 20$.

   C. $p_D = \frac{20}{3}$ and $p_G = \frac{25}{3}$.

   D. $p_D = 22$ and $p_G = 14$.

$MR = P(1 - \frac{1}{e})$

$= P(1 + \frac{1}{2}) = 10$

$P = \frac{20}{3}$

$MR = P(1 - \frac{1}{e})$

$= P(1 + \frac{1}{5})$

$P \cdot \frac{6}{5} = 10$

$P = \frac{50}{6}$

$= \frac{25}{3}$

8. The supply curve of apples is given by: $Q = c + dP$, where $c > 0$, Q is the quantity of apple supply (in million tons), and P is the price per unit of apple. If d equals zero, then the supply of apples is

   A. inelastic, but not perfectly inelastic.

   B. elastic, but not infinitely elastic.

   C. infinitely elastic.

   D. perfectly inelastic.

$Q = c$

ss

$e = \frac{\Delta Q}{\Delta P} = 0$

$e = \frac{dQ}{dP} \cdot \frac{P}{Q}$

$= 0 \cdot \frac{P}{Q}$

$= 0$

9. If two goods, X and Y, are perfect substitutes, which of the following assumptions about consumers' preferences is NOT satisfied?

   A. Completeness and transitivity

   B. Transitivity and reflexivity

   C. More is preferred to less

   D. Diminishing marginal rate of substitution

inc. = +ve

subs. = -ve

10. For a Giffen good, the income and substitution effects

A. work against each other.

B. work together.

C. can work together or in opposition to each other depending on preferences.

D. always exactly cancel each other.

11. Let $X \in \{0, 1, 2\}$ be a discrete random variable with pmf

$$f(x) = \begin{cases} 0.25 & \text{if } x = 0 \\ 0.5 & \text{if } x = 1 \\ 0.25 & \text{if } x = 2 \end{cases}$$

$y = \begin{cases} 0.25 \\ 0.5 \\ 0.25 \end{cases}$

Let $Y = (X - 1)^2$. Consider the following two statements:
Statement I: Y follows a Uniform distribution.
Statement II: Y follows a Bernoulli distribution.

A. Only Statement I is correct.

B. Only Statement II is correct.

C C. Both Statements I and II are correct.

D. None of the Statements are correct.

$p(x) = p^x q^{1-x}$

C

$\frac{6}{\mu}$

12. The mean and variance of a random variable are given by 7 and 49, respectively. The coefficient of variation for the random variable is

$\frac{\bar{X}}{V(X)} \times 100$

$\frac{1}{7} \times 100$

A. $\frac{1}{7}$.

B. -1.

C. 7.

D. 1.

13. $X_1, X_2, \cdots, X_{10}$ are independently and identically distributed as Bernoulli random variables with a probability of success p. The random variable Y is defined as,

$$Y = \sum_{i=1}^{10} X_i$$

$p^{x_1} q^{1-x_1} + p^{x_2} q^{1-x_2} + p^{x_3} q^{1-x_3} + \cdots$

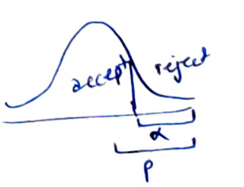
will follow

A. the same Bernoulli distribution with a probability of success p.

B. a Bernoulli distribution but with a different parameter.

C. a Binomial distribution.

D. a Uniform distribution.



14. In testing a hypothesis, the p value signifies

A. the smallest level of significance for rejecting the Null Hypothesis.

B. the largest level of significance for rejecting the Null Hypothesis.

C. the smallest level of significance for accepting the Null Hypothesis.

D. the smallest level of significance for accepting the Null Hypothesis.

15. Which of the following describes the property of an unbiased estimator?

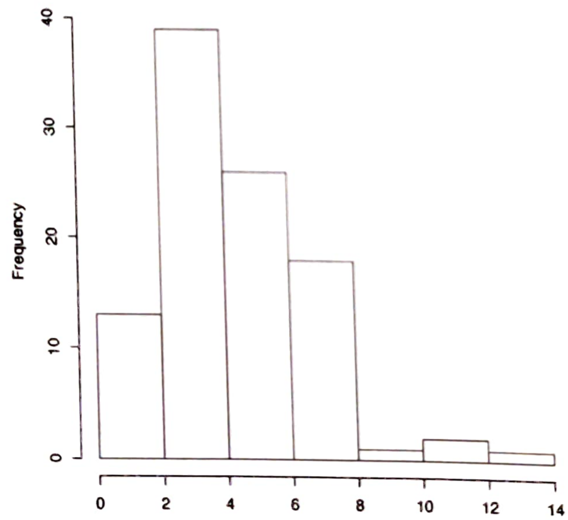
A. The shape of the sampling distribution of the estimator is approximately normal.

B. The mean of the sampling distribution of the estimator is found within one standard deviation away from the population parameter.

C. The mean of the sampling distribution of the estimator is equal to the population parameter being estimated.

D. The sampling distribution of the estimator has the smallest variation among all possible sampling distributions.

16. The histogram below depicts the frequency distribution of test scores for a randomly selected group of students. The relationship between the mean and median of the test scores is given by



A. Mean $>$ Median.

B. Mean $<$ Median.

C. Mean $\gtrless$ Median.

D. Mean $=$ Median.

17. Let P be a matrix such that

$$P = P^{\top}, \quad P^2 = P.$$

Then eigenvalues of this matrix can take only values

A. $\{1, 0\}$.

B. $\{0, -1\}$.

C. $\{1, 0, -1\}$.

D. none of the above.

18. Let X be a $N \times K$ ($N \geq K$) matrix which has full rank. Then the matrix $X^\top X$ is

K

A. invertible.

B. not invertible.

C. only invertible if $N = K$.

D. of indeterminate rank.

19. A function f is quasi-concave if for all x, y and $0 < \lambda < 1$

$$f\big(\lambda x + (1 - \lambda)y\big) \geq \min\big\{f(x), f(y)\big\},$$

and it is concave if for all x, y and $0 < \lambda < 1$

$$f\big(\lambda x + (1 - \lambda)y\big) \geq \lambda f(x) + (1 - \lambda)f(y).$$

Which of the following is TRUE?

A. Concavity implies quasi-concavity.

B. Quasi-concavity implies concavity.

C. There is no relationship between quasi-concavity and concavity.

D. None of the above.

20. Suppose $v_1, v_2, \ldots, v_n$ are n linearly independent vectors in some vector space. Let the vectors $w_1, w_2, \ldots, w_n$ be defined by $w_1 = v_1, w_2 = v_1 + v_2, w_3 = v_1 + v_2 + v_3, \ldots, w_n = v_1 + v_2 + \cdots + v_n$. Then,

A. strict nonempty subsets of $\{w_1, w_2, \ldots, w_n\}$ are linearly independent, but $\{w_1, w_2, \ldots, w_n\}$ are not linearly independent.

B. $\{w_1, w_2, \ldots, w_n\}$ may or may not be linearly dependent.

C. $\{w_1, w_2, \ldots, w_{n-1}\}$ are linearly dependent.

D. $\{w_1, w_2, \ldots, w_n\}$ are linearly independent.

$v_1, v_1+v_2, v_1+v_2+v_3$

$m_1 v_1 + m_2(v_1+v_2) + m_3(v_1+v_2+v_3) = 0$

$(m_1+m_2+m_3)v_1 + (m_2+m_3)v_2 + m_3 v_3 = 0.$

$m_3 = 0$

$m_1 + m_2 = 0$

$m_2 = 0.$

II Multiple Choice Questions (Two marks each)

21. Let the utility from consuming the bundle (x_1, x_2, x_3) is $u(x_1, x_2, x_3) = \text{median}\{x_1, x_2, x_3\}$. Let (p_1, p_2, p_3) with $p_i > 0, i \in \{1, 2, 3\}$ be the price vector and income $I > 0$, then the number of utility-maximizing bundles

    A. is at most one.

    B. is at most two.

    C. is at most three.

    D. is at most four.

$\left(\frac{I}{p_1}, 0, 0\right)$ $\left(\frac{I}{p_1+p_2}, \frac{I}{p_1+p_2}, 0\right)$ ✓

$\left(0, \frac{I}{p_2}, 0\right)$

3

22. The government is planning to build an interstate highway between two states, named A and B. The highway costs $C > 0$ to the government, and the values of the highway to the states A and B are $v_A \geq 0$ and $v_B \geq 0$, respectively; $v_A < C$, $v_B < C$, and $v_A + v_B > C$. Each state $i \in \{A, B\}$ simultaneously bids $b_i \in [0, \infty)$. If $b_A + b_B \geq C$, the highway is constructed. For any distinct $i, j \in \{A, B\}$, state i pays $C - b_j$ to the government if $b_j < C \leq b_A + b_B$. (There is no payment otherwise.) The payoff of a state is the value of the highway to the state minus its own payment to the government if the highway is built, and 0 otherwise.

    A. In Nash equilibrium, each state i will necessarily bid less than v_i.

    B. In Nash equilibrium, each state i will bid exactly v_i.

    C. There are two equilibria, $(v_1, 0)$ and $(0, v_2)$.

    D. None of the above.

23. A worker has utility over consumption c and leisure l given by

$$U(c, l) = \alpha \frac{c^\delta}{\delta} + \beta \frac{l^\delta}{\delta}$$

where $0 < \delta < 1$. She has T hours to allocate between leisure and work. For each hour she works, she earns a wage of w to spend on consumption c. The price of c is 1. She also receives an additional 'non-labor income' m regardless of how much she works. She maximizes utility subject to the following constraints:

$$c \leq w(T - l) + m$$
$$c \geq 0$$
$$0 \leq l \leq T$$

Assume interior solution, then

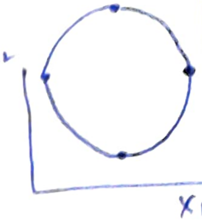
A. c is normal and l is inferior good.

B. c is inferior and l is inferior good.

C. c is inferior and l is normal good.

D. c is normal and l is normal good.

$y = l^{\frac{1}{2}} k^{\frac{1}{2}}$ $MP_L = \frac{1}{2}\frac{1}{\sqrt{L}} \cdot k^{\frac{1}{2}}$ $AP_L = \frac{1}{2}\sqrt{\frac{K}{L}}$



24. For a production function $y = f(x_1, x_2)$ homogeneous of degree one in x_1 and x_2,

   A. if the average product for x_1 is increasing in x_1, it implies that the marginal product of the other input x_2 is negative.

   B. if the marginal product for x_1 is increasing in x_1, it implies that the average product of the other input x_2 is negative.

   C. if the average product for x_1 is decreasing in x_1, it implies that the marginal product of the other input x_2 is negative.

   ~~D.~~ if the marginal product for x_1 is increasing in x_1, it implies that the marginal product of the other input x_2 is negative.

25. Consider an exchange economy with two agents, 1 and 2, and two goods, X and Y. Agent 1's endowment is $(4, 0)$ and Agent 2's endowment is $(0, 1)$. Agent 1's utility function $u_1(x_1, y_1) = x_1^{1/3} y_1^{2/3}$ and Agent 2's utility function $u_2(x_2, y_2) = x_2^{2/3} y_2^{1/3}$. The price of good X is normalized to equal 1. In a competitive equilibrium, the price of good Y equals

   A. 1

   B. 2

   C. 3

   ~~D.~~ 4

$\frac{2}{3}\cdot\frac{m_1}{p_2} + \frac{1}{3}\cdot\frac{m_2}{p_2} = 1.$ $\frac{1}{3}\frac{m_1}{p_1} + \frac{2}{3}\frac{m_2}{p_1} = 4.$

$\frac{2}{3}\cdot\frac{4p_1}{p_2} + \frac{1}{3}\cdot\frac{p_2}{p_2} = 1$ $\frac{1}{3}\frac{(4p_1)}{p_1} + \frac{2}{3}\left(\frac{p_2}{p_1}\right) = 4$

$\frac{2}{3}\cdot\frac{4 \cdot p_1}{p_2} = \frac{2}{3} = 1.$ $\frac{4}{3} + \frac{2}{3}\frac{p_2}{1} = 4$

$\frac{1}{p_2} = \frac{1}{4} \Rightarrow p_2 = 4.$ $\frac{2}{3}p_2 = 4 - \frac{4}{3} = \frac{8}{3}$ $p_2 = 4$

26. Consider an exchange economy with two agents, 1 and 2, and two goods, X and Y. Agent 1's endowment is $(k, 0)$, where k is some positive real number, and Agent 2's endowment is $(0, 1)$. Agent 1's utility function $u_1(x_1, y_1) = x_1^{1/3} y_1^{2/3}$ and Agent 2's utility function $u_2(x_2, y_2) = x_2^{2/3} y_2^{1/3}$. The price of good X is normalized to equal 1. Let p be the price of good Y in competitive equilibrium. Then, the derivative $\frac{dp}{dk}$ equals

   $\frac{1}{3}\frac{m_1}{p_1} + \frac{2}{3}\cdot\frac{m_2}{p+1} = k$ $\left(k - \frac{k}{3}\right)\cdot\frac{3}{2}$ $= 1.$ $p = \left(k - \frac{k}{3}\right)\cdot\frac{3}{2}$ $p = \frac{12k - k}{2}$

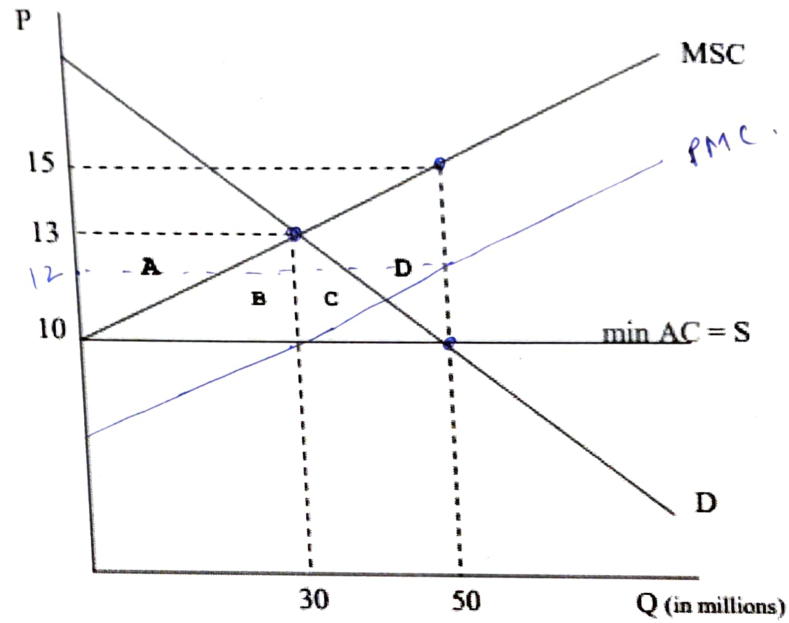
   A. 2

   ~~B. 1~~

   C. 1.5

   D. 3

[The following two questions are based on the problem and the graph outlined below.]
Coal production creates air pollution as a byproduct which is bad for health. Assume that the coal production industry is perfectly competitive and the minimum Average Cost (min AC) of firms is Rs. 10. The market demand is linear, and without any government intervention, the market output will be 50 million units at a price of Rs. 10. The marginal social cost (given by the MSC curve) of producing coal is Rs. 15 at this level of output. The socially optimal output is 30 million units at a price of Rs. 13. This output can be attained by levying an excise tax of Rs. 3 on the production of each unit of coal.



27. The loss of consumer surplus associated with the Rs. 3 tax when the market moves to the socially optimal output level is equal to

A. Rs. 130 million.

B. Rs. 100 million.

~~C~~. Rs. 120 million.

D. Rs. 90 million.

(13−10) 30 · 90 + ½ × 3 × 20 · 120

28. What is the reduction in pollution costs after the tax of Rs. 3 on each unit of coal?

A. Rs. 70 million.

~~B~~. Rs. 80 million.

C. Rs. 90 million.

D. Rs. 100 million.

½ × 20 × 5 + C(30) 80

29. Consider a function $f : \Re^2 \rightarrow \Re$. Suppose, for every $p \in \Re^2$, there exists $x(p) \in \Re^2$ such that $f(x(p)) \geq 1$ and $p.x(p) \leq p.y$ for every $y \in \Re^2$ such that $f(y) \geq 1$. Define $g : \Re^2 \rightarrow \Re$ by $g(p) = p.x(p)$. Then, g is

A. linear.

~~B~~. convex.

C. quasi-convex.

~~D~~. concave.

30. The function defined by $f(x) = x^5 + 7x^3 + 13x - 18$

$(x + x^3 + x^5)(6)(x^3 + 2x - 3)$

$x((x^2)^2 - 1) \cdot 6$ / $x^2 - 1$

$x^3 + 2x - 3 = 0$

A. may have 5 distinct real roots.

B. has no real root.

~~C.~~ has 3 real roots.

D. has exactly 1 real root.

31. Suppose $A_0, A_1, A_2, \cdots$ is a countable collection of subsets of real numbers. If the real number x satisfies $x \in \cap_{n=0}^{\infty} \cup_{k=n}^{\infty} A_k$, then

A. x belongs to every A_k, $k = 0, 1, 2, \cdots$.

B. x belongs to all but a finite number of the sets $A_k, k = 0, 1, 2, \cdots$.

C. x belongs to infinitely many of the sets $A_k, k = 0, 1, 2, \cdots$.

D. x belongs to finitely many of $A_k, k = 0, 1, 2, \cdots$.

32. Let A, B be two distinct $n \times n$ square matrices such that $AB = BA$ and $A^2 = B^2$. Then the determinant of $A + B$ is equal to

A. 3.

B. 1.

~~C.~~ 0.

D. 2.

2A or 2B.

$(A+B)^2 = A^2+B^2+2AB.$
$= 2A^2+2AB.$
$= 2A(A+B). = (A+B)^2$
$= 2B(A+B)$

$2A = A+B.$
$A=B.$

33. Consider the function $f : (0, 1] \to \Re$ defined by

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational} \\ 1/q & \text{if } x = p/q \text{ in lowest terms.} \end{cases}$$

Consider the following statements:
Statement I: f is discontinuous at all rational numbers in $(0, 1]$.
Statement II: f is continuous at all irrational numbers in $(0, 1]$.

~~A.~~ Statement I is TRUE and Statement II is FALSE.

B. Both Statements are TRUE.

C. Statement I is FALSE and Statement II is TRUE.

~~D.~~ Both Statements are FALSE.

$\frac{irr}{0} \quad \frac{1}{2} \quad \frac{irr}{0}$

$P^T.P.A.P^T.Q^{11}.P$
$A^1.P^T.Q^{11}P$
A^{12}

34. The matrix $Q = PAP^T$, where P^T is the transpose of the matrix P, and

$$P = \begin{pmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$A^2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

Then, $P^TQ^{12}P$ equals

~~A.~~

$$\begin{pmatrix} 1 & 12 \\ 0 & 1 \end{pmatrix}$$

B.

$$\begin{pmatrix} 1 & 144 \\ 0 & 1 \end{pmatrix}$$

C.

$$\begin{pmatrix} 1 & 0 \\ 144 & 1 \end{pmatrix}$$

D.

$$\begin{pmatrix} 2+\sqrt{3} & 1 \\ -1 & 2-\sqrt{3} \end{pmatrix}$$

35. Let

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{matrix} a \\ 2a+b \\ 3a+2b+c \end{matrix}$$

and B_1, B_2, B_3 be three 3×1 column vectors, such that,

$$\begin{matrix} 1 \\ -2 \\ 1 \end{matrix} \; AB_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, AB_2 = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}, AB_3 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

$2+b=0.$
$b=-2$
$3-4+1=0$

Let B be the 3×3 matrix whose 3 columns are B_1, B_2 and B_3 respectively. Then the determinant $det(B)$ is equal to

A. -3.
~~B.~~ 3.
C. $-\frac{3}{2}$.
D. $\frac{3}{2}$.

$$\begin{vmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{vmatrix}$$

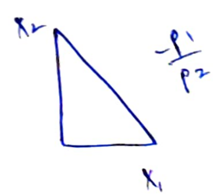
$= 1(3-4) - 2(6+1) + 2(8+1)$
$= -1 - 14 + 18$
$= 3$

36. For any $p_1, p_2 \geq 0$ and finite I, we define

$$B(p_1, p_2, I) = \{(x_1, x_2) : x_1 \geq 0, x_2 \geq 0, p_1x_1 + p_2x_2 \leq I\}.$$

Then the set $B(p_1, p_2, I)$ is always

~~A.~~ Closed.
B. Bounded.
C. Compact.
D. none of the above.



37. A function $f : \Re \to \Re$ is called quasi-concave if for all x, y and $0 < \lambda < 1$

$$f\left(\lambda x + (1-\lambda)y\right) \geq \min\left\{f(x), f(y)\right\}.$$

Then the following holds for quasi-concave functions.

A. If f is non-decreasing, it is quasi-concave.

B. If f is non-increasing, it is quasi-concave.

C. There exists a number x^* such that f is non-decreasing on $\{x \leq x^*\}$ and non-increasing on $\{x > x^*\}$.

D. All of the above.

38. Let set $A = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$, where $\mathbb{N}$ is the set of natural numbers not containing 0. Then Set A is

A. Closed.

B. Open.

C. neither Closed nor Open.

D. none of the above.

39. Suppose A is a matrix such that there exist nonzero vectors v and w with

$$Av = 3w, \quad Aw = 3v.$$

Then the eigenvalues of the matrix A are

$(A-\lambda)I = 0$

$(A-3)I \to 0$

$I = +1 = v - w$

A. $\{1, -1\}$.

B. $\{\sqrt{3}, -\sqrt{3}\}$.

C. $\{3, -3\}$.

D. none of the above.

40. Let $d\left((x_1, x_2), (y_1, y_2)\right) = \max\left\{|x_1 - y_1|, |x_2 - y_2|\right\}$ be the distance between two points (x_1, x_2) and (y_1, y_2). Then the locus of points at a unit distance from the origin is

A. a square with side length $= 1$.

B. a square with side length $= \sqrt{2}$.

C. a square with side length $= 2$.

D. a circle with radius $= 1$.

41. Let $f : R \to R$ such that $0 < f'(x) \leq 1$ for all $x \geq 1$. Suppose $f(1) = 2$. It follows that for all $x \geq 10$, $f(x) \leq$

$0 < \frac{f(x) - 2}{x - 1} \leq 1.$

A. $1 + \frac{1}{2}x$

$f(x) \leq x - 1 + 2$

B. $\frac{1}{2} + x$

$f(x) \leq x + 1$

C. $1 + x$

196
−14
182

14
13

D. $3 - x$

42. Consider the following real-valued function, where a, b are real numbers and n is a positive integer:

$$f(x) = \begin{cases} b & if\ x < a \\ b + (x-a)^n & if\ x \geq a \end{cases}$$

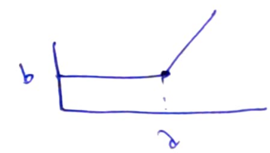
This function has a kink at $x = a$ if

A. $n = 4$.

B. $n = 3$.

C. $n = 2$.

D. $n = 1$.



43. The price and quantity of two goods in two periods in an economy are given below.
Good 1: $p_t^1 = 1$, $p_{t+1}^1 = 1$, $q_t^1 = 1$, $q_{t+1}^1 = 1.1$
Good 2: $p_t^2 = 1$, $p_{t+1}^2 = 1.4$, $q_t^2 = 1$, $q_{t+1}^2 = 1.3$
Here, p_i^j is the price of good $j = \{1, 2\}$ at time $i = \{t, t+1\}$; and similarly, q_i^j is the quantity of good $j = \{1, 2\}$ at time $i = \{t, t+1\}$. The growth rate of real GDP in this economy using the chain index is given by (with nearest integer approximation)

A. 19%.

B. 21%.

C. 20%.

D. 22%.

$1 + 1 = 2$
$1.1 + 1.4 \times 1.3 = 2.92$
$1.1 + 1.3 = 2.4$

44. Consider a labour market characterized by the following production and labour supply functions:

$$F(N) = 20N - N^2$$
$$N_s = \frac{1}{2}\frac{w}{p}$$

$MP_L = dd_L = 20 - 2N$
$SS_L = \frac{1}{2}\frac{w}{p}$

Here, N_s is the labour supply, w is the nominal wage and N is the labour input. Take prices p as given. Consider the case in which the government introduces a minimum nominal wage $\bar{w} = 50$. The equilibrium real wage under the case when $\frac{5}{2} < p < 5$ is given by

2.5 5

A. $\frac{50}{p}$

B. $\frac{25}{p}$

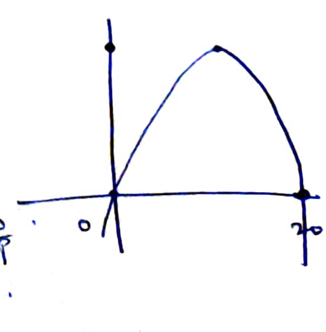
C. $\frac{3}{2p}$

D. $\frac{50}{p^2}$

$MP_L = 20 - 2N = W = 2p(N)$

$20 - (2 + 2p) N = 0$

$2W = 20 - 50$



$20 - 2N = \frac{w}{p}$

$N = (20 - \frac{w}{p}) \frac{1}{2}$

$N = 10 - \frac{25}{p}$

$\frac{1}{2}\frac{w}{p} = 10 - \frac{1}{2}\frac{w}{p}$

$\frac{w}{p} = 10$

45. Consider the following IS-LM model with prices fixed at $P = 1$ (consider short run analysis). Notations are all standard.

$$\begin{aligned}
\frac{M^d}{P} &= Y - r \\
C &= 1 + 0.5Y \\
I &= 1 - 0.5r \\
G &= \bar{G} \\
Y &= C + I + G \\
\frac{M^s}{P} &= \frac{\bar{M}}{P} \\
\frac{M^d}{P} &\leq \frac{M^s}{P}, \text{ with } \frac{M^d}{P} = \frac{M^s}{P} \text{ if } r > 0 \\
r &= i - \pi^e \\
\pi^e &= 0
\end{aligned}$$

$Y = 1 + 0.5Y + 1 - 0.5r + \bar{G}$

$0.5Y = 2 - 0.5r + \bar{G} \rightarrow Y = 4 - r + 2\bar{G}$

$\frac{\bar{M}}{P} = Y - r = 4 - r + 2\bar{G} - r$

$\frac{\bar{M}}{P} = 4 + 2\bar{G} - 2r.$

$r = \frac{\frac{\bar{M}}{P} - 4 - 2\bar{G}}{2}$

$Y = 4 - \left(\frac{\frac{\bar{M}}{P} - 4 - 2\bar{G}}{2}\right) + 2\bar{G}$

$Y = 6 - \frac{\bar{M}}{2P} + 3\bar{G}$

$r = \frac{4 + 2\bar{G} - \frac{\bar{M}}{P}}{2}$

$Y = \frac{\bar{M}}{P} + r = \left(\frac{1}{2}\right)\frac{\bar{M}}{P} + 2 + \bar{G}$

$\Delta Y > 0.$

Suppose that the economy described above is going through a recession and the government is trying to stimulate the economy. The condition under which the monetary policy will be effective in stimulating the economy is given by

$Y = 6 - \frac{\bar{M}}{2P} + 3\bar{G}$

$\frac{\Delta Y}{\Delta \frac{\bar{M}}{2P}} > 0 = 0 - \frac{1}{2} + 3\bar{G}$

A. $\bar{M} < 4 + 2\bar{G}$.

B. $\frac{\bar{M}}{2} = 4 + 2\bar{G}$.

C. $\bar{M} > 4 + 2\bar{G}$.

D. $\frac{\bar{M}}{2} < 4 + 2\bar{G}$.

46. Nicholas Kaldor in his famous 1961 paper, described a list of six "stylized" facts about economic growth. Which of the following facts do not belong to his original list.

Fact 1. Per capita output grows over time, and its growth rate does not tend to diminish.
Fact 2. Physical capital per worker grows over time.
Fact 3. The rate of return to capital is nearly constant.
Fact 4. The ratio of physical capital to output is nearly constant.
Fact 5. The share of labour earnings in national income is decreasing and share of physical capital earnings in national income is increasing.
Fact 6. The growth rate of output per worker differs across countries.

A. Fact 3.

B. Fact 5.

C. Fact 6.

D. Fact 2.

47. Suppose the period-t utility function, u_t, is given by $u_t = ln\ c_t + b\ \frac{(1-l_t)^{1-\gamma}}{1-\gamma}$, $b > 0$, $\gamma > 0$. The functional form of this utility function when $\gamma = 1$, is given by

$MRS = \frac{\frac{1}{c_t}}{\frac{b\cdot(1-\gamma)\cdot(1-l_t)^{-\gamma}}{1-\gamma}} = -\frac{1}{b}\cdot\frac{(1-l_t)^{\gamma}}{c_t}$

A. $u_t = ln\ c_t + \frac{b}{1-l_t}$.

B. $u_t = ln\ c_t + ln\ b$.

C. $u_t = ln\ c_t + b\ ln(1 - l_t)$.

$\frac{\frac{1}{c_t}}{b \cdot \frac{1}{1-l_t}(-1)}$

D. $u_t = ln\ c_t + b$.

48. The national income in a closed economy is given by $Y = C + I + G$ where variables have their standard meaning. Assume that the consumption is given by $C = c_0 + c_1(Y - T)$ where $c_0 > 0$, $c_1 > 0$ and $Y - T$ is the income net of taxes. In this framework, suppose that the government introduces a new spending program that is fully paid for by an increase in taxes such that $dY = dG$.
Statement A: Such a program will have no effect on investment.
Statement B: The Keynesian multiplier in this framework is less than unity. $= 1.$

   A. Statement A is FALSE and Statement B is TRUE.

   B. Statement A is TRUE and Statement B is FALSE.

   C. Both Statements are TRUE.

   D. Both Statements are FALSE.

$I = \bar{I} - bi.$

$T\uparrow \Rightarrow G\uparrow$ equal.

$\Rightarrow C\downarrow (-G).$

49. The value of R^2 from the simple linear regression below

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

is given by 0.49. What is the correlation coefficient between the dependent and the independent variable?

   A. 0.49.

   B. $\sqrt{0.49}$.

   C. 0.49^2.

   D. $(1 - \sqrt{0.49})$.

$b_{yx} = r = \sqrt{R^2}.$

50. During the COVID-19 pandemic, a hospital in a residential college found that any randomly chosen student in a hostel is 80% likely to have COVID. Furthermore, if a student is COVID-positive, there is a 10% chance that the RT-PCR test will show a negative result. Conversely, even if a student does not have the virus, there is a 7% chance that the RT-PCR test will be positive. What is the probability that a student is actually COVID-positive, given a positive RT-PCR report?

$E_1 = 80$

$E_2 = 20$

$P/E_1 = 90$

$P/E_2 = 7$

   A. 0.98.

   B. 0.92 .

   C. 0.72.

   D. 0.84.

$$\frac{80 \times 90}{(20 \times 7) + (80 \times 90)}$$

$$\frac{7200}{140 + 7200} \quad \frac{7200}{7340}$$

51. Out of an incoming batch of 40 students at the M.Sc. Economics program at an institute, 20 of them studied economics as a major subject in their undergraduate. Four students are randomly selected from the batch. What is the probability that two of them will have a non-economics background?

   A. $\frac{1}{2}$.

   B. $\frac{3}{8}$.

$p = \frac{1}{2}.$

$P(X=2) = \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^2$

$$\frac{20C_2 \times 20C_2}{40C_4}$$

$$= \frac{\frac{20!\,20!}{2!\,18!\,2!\,18!}}{40!/4!\,36!}$$

$$\frac{(20 \times 19)^2}{\frac{40 \times 39 \times 38 \times 37}{4 \times 3 \times 2 \times 1}}$$

$$\frac{36100}{91390} = 0.395$$

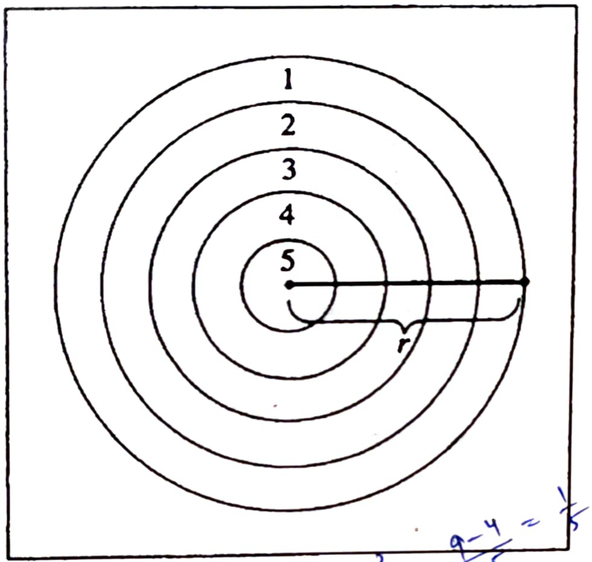
C. $\left(\frac{1}{2}\right)^4$.

D. $\frac{5}{8}$.

$${}^4C_2 \cdot \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 \qquad \frac{12}{25} = \frac{3}{2^3} = \frac{3}{8} \checkmark$$

$$\frac{4\times 3}{2}\cdot\left(\frac{1}{2}\right)^4$$

52. The game of darts is played by throwing a dart at a board and receiving a score corresponding to the number assigned to the region in which it lands (as shown in the figure below). Assume that the probability of the dart hitting a particular region is proportional to the region's area. The dart board with radius r has five regions, and the distance between the rings is $r/5$. If we further assume that a dart always hits one of the numbered regions, what is the probability that a player will score 3 points after throwing a dart?



$$\left(\frac{r}{5}\right)^2 + \left(\frac{2r}{5}\right)^2 + \left(\frac{3r}{5}\right)^2 + \left(\frac{4r}{5}\right)^2 + (r)^2 = 1.$$

$$r^2\frac{(1+4+9+16+25)}{25}$$

$$r^2 = \frac{1.25}{55}$$

$$\frac{6}{5}r^2 + r^2 = 1. \qquad \frac{11}{5}r^2 = 1. \qquad r^2 = \frac{5}{11}.$$

$$P = \frac{5}{11}\cdot\left(\frac{3}{5}\right)^2 = \frac{9}{55}$$

A. $\frac{1}{25}$.

B. $\frac{3}{25}$.

~~C.~~ $\frac{5}{25}$.

D. $\frac{9}{25}$.

$$P(3) = \left(\frac{3r}{5}\right)^2 - \left(\frac{2r}{5}\right)^2 = \frac{9-4}{25} = \frac{1}{5}$$

$$\Sigma P = 1 + 2 + 3 + 4 + 5$$

$$= (1-2) + (2-3) + (3-4) + (4-5) + 5$$

$$= 1. \quad = \pi r^2 = 1. \quad r = 1.$$

$$P = \frac{3}{5}\cdot\frac{25}{55} \qquad \frac{9}{25}\cdot\frac{25}{55} \qquad \frac{3}{55}$$

53. For the function below

$$f(x,y) = \begin{cases} cx^2y & \text{if } x^2 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

to be the probability density for two jointly distributed random variables X and Y, the value of c must be

A. 25/4

B. 1/3

C. 2/3

D. 21/4

$$\int_0^y czy \cdot dz \qquad cy\cdot\frac{z^2}{2} = \frac{cy^3}{2}$$

$$\int_y\int_x cx^2y \cdot dx\,dy = 1.$$

$$cx^2\cdot\frac{y^2}{2}\Big|_{x^2}^{1} = \int_0^x cx^2\left(\frac{1}{2} - \frac{x^2}{2}\right)\cdot dx$$

$$\int_0^x cz\left(\frac{1}{2} - \frac{z}{2}\right)\cdot dz$$

54. In a survey of 1055 voters were asked if they prefer candidate A over B. Let p and $\hat{p}$ denote the fraction of voters in the population and the sample that prefers voter A,

β= accept when false.
power of test = 1-β

$\sqrt{\frac{p(1-p)}{n}}$

respectively. You want to test the hypothesis, $H_o : p = 0.5$, against the alternative $H_A : p \neq 0.5$. You are considering using a test criteria that will reject the H_0 if $|\hat{p} - 0.5| > 0.02$. The power of the test is

$|\hat{p}-0.5| < 0.02$
$\hat{p} < 0.52$
$\hat{p} > 0.48$

A. 0.09.

B. 0.15.

C. 0.20.

D. 0.25.

$1.96 = \frac{\hat{p}-0.5}{0.0154}$
0.530184
0.469816

55. A survey recorded the weights (x_i) and heights (y_i) of 300 students. The resulting summary statistics are below

sample mean of weights	$\bar{x} = 60$
sample mean of heights	$\bar{y} = 170$
sample standard deviation of heights	$s_x = 10$
sample standard deviation of weights	$s_y = 20$
sample correlation coefficient between heights and weights	$\rho_{xy} = 0.5$

A research scholar is interested in estimating the following simple regression model using the Ordinary Least Square method

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i.$$

Given the summary statistics, the estimates of the parameters of the model are

A. $\hat{\beta}_0 = 100; \hat{\beta}_1 = \frac{7}{6}$.

B. $\hat{\beta}_0 = 110; \hat{\beta}_1 = 1$.

C. $\hat{\beta}_0 = 120; \hat{\beta}_1 = \frac{5}{6}$.

D. $\hat{\beta}_0 = 130; \hat{\beta}_1 = \frac{2}{3}$.

$\rho_{xy} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{S_x \cdot S_y}$

$b_{xy} = \rho_{xy} \cdot \frac{S_y}{S_x}$

$\Sigma xy = 0.5 \times 10 \times 20$
$= 5 \times 20$
$= 100$

$170 - 60 \times 1 = 110$ ✓

56. Consider the following two population regression lines where the dependent and the independent variables are flipped

$$y_i = \overset{-108}{\beta_0 + \beta_1 x_i} + \epsilon_i \tag{1}$$

$$x_i = \overset{0.006}{\alpha_0 + \alpha_1 y_i} + \nu_i \tag{2}$$

The estimated slope coefficients from equations (1) and (2) are -108 and 0.006, respectively. Which of the following statements is/are correct?

A. R^2 from equation (1) is 0.648.

B. R^2 from equation (2) is 0.648.

C. The correlation between x and y is 0.805

D. All of the above statements are correct.

$R^2 = b_{xy} b_{yx}$.

57. Let X and Y be two independent random variables. Define $[X \wedge Y] = \min(X, Y)$ and $[X \vee Y] = \max(X, Y)$. The sum of the expected values of the random variables, $E[X] + E[Y]$ is given by

$E(\wedge) = 1/3$
$E(\vee) = 2/3$

$E(X \wedge Y) = E(X \geq z, \text{and } \geq z)$
$= E(X \geq z) . E(Y \geq z).$

$E(X \vee Y) =$

A. $E[X \vee Y] + E[X \wedge Y]$.

B. $E[X \vee Y] - E[X \wedge Y]$.

C. $\frac{1}{2}\Big[E[X \vee Y] + E[X \wedge Y]\Big]$.

D. None of the above.

58. Consider the following population regression equation

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i \tag{3}$$

The estimators of the unknown parameters using an Ordinary Least Square model are $\hat{\beta}_0$ and $\hat{\beta}_1$. Which of the following statements is/are correct?

A. The parameters β_0 and β_1 follow a normal distribution when the sample size is large.

B. ex-ante $\hat{\beta}_0$ and $\hat{\beta}_1$ are random variables.

C. The parameters cannot be estimated unless ϵ_i is homoscedastic.

D. All of the above statements are correct.

59. Assume that each of n farmers can costlessly produce any amount of rice (in kg). Suppose that the k^{th} farmer produces W_k kg; so that the total amount of rice produced is $W = W_1 + W_2 + \cdots + W_n$. The price P at which rice sells is determined by the demand $P = e^{-W}$. The strategy of producing rice that strongly dominates all other quantities by the k^{th} profit-maximizing farmer is given by

$P . W_k = R$

$e^{-W} . W_k = R$

A. $W_k = 10$

B. $W_k = 1$

C. $W_k = 0.1$

D. $W_k = \infty$

60. Players 1 and 2 are playing the "stag hunt" game, and both have two strategies, S and H. The payoff matrix below summarizes their payoffs. There are ____ Pure Strategy Nash Equilibria of the game.

$\frac{dR}{dW_k} = e^{-W} . (-1) . W_k + e^{-W} . 1 = 0$

$e^{-W} = e^{-W} . W_k$

		Player 2	
		S	H
Player 1	S	$(3, 3)$	$(0, 1)$
	H	$(1, 0)$	$(1, 1)$

A. one

B. two

C. three

D. countably infinite

$\Sigma\, x . P(x)$